

9.2 Punten, lijnen en afstanden.

Opgave 21:

$$x_B - x_A = 7 - 1 = 6$$

$$y_B - y_A = 5 - 2 = 3$$

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$$

Opgave 22:

a. $d(A, B) = \sqrt{(5 - (-3))^2 + (-2 - 4)^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

b. $\sqrt{(x+3)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y+2)^2}$

$$(x+3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2$$

$$x^2 + 6x + 9 + y^2 - 8y + 16 = x^2 - 10x + 25 + y^2 + 4y + 4$$

$$16x - 12y = 4$$

$$4x - 3y = 1$$

Opgave 23:

a. $\sqrt{(x+1)^2 + (y+1)^2} = \sqrt{(x-1)^2 + (y-5)^2}$

$$(x+1)^2 + (y+1)^2 = (x-1)^2 + (y-5)^2$$

$$x^2 + 2x + 1 + y^2 + 2y + 1 = x^2 - 2x + 1 + y^2 - 10y + 25$$

$$4x + 12y = 24$$

$$m: x + 3y = 6$$

$$\sqrt{(x-7)^2 + (y+1)^2} = \sqrt{(x-1)^2 + (y-5)^2}$$

$$(x-7)^2 + (y+1)^2 = (x-1)^2 + (y-5)^2$$

$$x^2 - 14x + 49 + y^2 + 2y + 1 = x^2 - 2x + 1 + y^2 - 10y + 25$$

$$-12x + 12y = -24$$

$$n: x - y = 2$$

b.
$$\begin{cases} x + 3y = 6 \\ x - y = 2 \end{cases} -$$

$$4y = 4$$

$$y = 1$$

$$x = 3$$

$$S(3,1)$$

c. punt S is het middelpunt van de omschreven cirkel van $\triangle ABC$

Opgave 24:

lijn m is de middelloodlijn van AB

$$\sqrt{(x+1)^2 + (y+3)^2} = \sqrt{(x-7)^2 + (y-1)^2}$$

$$(x+1)^2 + (y+3)^2 = (x-7)^2 + (y-1)^2$$

$$x^2 + 2x + 1 + y^2 + 6y + 9 = x^2 - 14x + 49 + y^2 - 2y + 1$$

$$16x + 8y = 40$$

$$m: 2x + y = 5$$

lijn n is de middelloodlijn van AC

$$\sqrt{(x+1)^2 + (y+3)^2} = \sqrt{(x+2)^2 + (y-4)^2}$$

$$(x+1)^2 + (y+3)^2 = (x+2)^2 + (y-4)^2$$

$$x^2 + 2x + 1 + y^2 + 6y + 9 = x^2 + 4x + 4 + y^2 - 8y + 16$$

$$-2x + 14y = 10$$

$$n: -x + 7y = 5$$

punt S is het snijpunt van de lijnen m en n

$$\begin{cases} 2x + y = 5 & \times 1 \\ -x + 7y = 5 & \times 2 \end{cases}$$

$$\begin{cases} 2x + y = 5 \\ -2x + 14y = 10 & + \end{cases}$$

$$\begin{cases} 2x + y = 5 \\ -2x + 14y = 10 & + \end{cases}$$

$$\begin{cases} 2x + y = 5 \\ -2x + 14y = 10 & + \end{cases}$$

$$15y = 15$$

$$y = 1$$

$$x = 2$$

$$S(2,1)$$

$$d(A, S) = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

Opgave 25:

a. je cirkelt vanuit $M(1,4)$ een touwtje om van lengte 5 dus ontstaat een cirkel met straal 5

$$b. (x-1)^2 + (y-4)^2 = 100$$

$$c. d(M, O) = \sqrt{1^2 + 4^2} = \sqrt{17}$$

$$(x-1)^2 + (y-4)^2 = 17$$

$$d. M(-2,3) \text{ en } r = \sqrt{5}$$

Opgave 26:

$$a. d(A, M) = \sqrt{5^2 + 1^2} = \sqrt{26}$$

$$(x+2)^2 + (y-1)^2 = 26$$

$$b. d(N, x-as) = 2$$

$$(x-4)^2 + (y+2)^2 = 4$$

c. lijn m is de middelloodlijn van PQ

$$\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-7)^2 + (y-2)^2}$$

$$(x-1)^2 + (y-2)^2 = (x-7)^2 + (y-2)^2$$

$$x^2 - 2x + 1 = x^2 - 14x + 49$$

$$12x = 48$$

$$x = 4$$

lijn n is de middelloodlijn van PR

$$\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y-6)^2}$$

$$(x-1)^2 + (y-2)^2 = (x-3)^2 + (y-6)^2$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = x^2 - 6x + 9 + y^2 - 12y + 36$$

$$4x + 8y = 40$$

$$x + 2y = 10$$

punt M is het snijpunt van de lijnen m en n

$$\begin{cases} x = 4 \\ x + 2y = 10 \end{cases}$$

$$4 + 2y = 10$$

$$2y = 6$$

$$y = 3$$

$$M(4,3)$$

$$d(M, P) = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$(x-4)^2 + (y-3)^2 = 10$$

Opgave 27:

a. $A(2,0)$ $B(0,4)$

$$AB = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$$

b. $Opp(\Delta OAB) = \frac{1}{2} \cdot OA \cdot OB = \frac{1}{2} \cdot AB \cdot OC$

dus $OA \cdot OB = AB \cdot OC$

$$OA = 2 \quad OB = 4$$

$$2 \cdot 4 = 2\sqrt{5} \cdot OC$$

$$OC = \frac{8}{2\sqrt{5}} = \frac{4}{\sqrt{5}} = \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{5} = \frac{4}{5}\sqrt{5}$$

Opgave 28:

lijn l : $\frac{x}{2} + \frac{y}{4} = c$ door (x_P, y_P)

$$\frac{x}{2} + \frac{y}{4} = \frac{x_P}{2} + \frac{y_P}{4}$$

Opgave 29:

lijn k : $3x + 4y = 24$

$P(-2, -1)$ dus lijn l : $3x + 4y = -10$

$$d(P, k) = \frac{|3 \cdot -2 + 4 \cdot -1 - 24|}{\sqrt{3^2 + 4^2}} = \frac{|-34|}{\sqrt{25}} = \frac{34}{5}$$

$$d(O, k) = \frac{|24|}{\sqrt{25}} = \frac{24}{5}$$

$$d(O, l) = \frac{|10|}{\sqrt{25}} = \frac{10}{5}$$

dus $d(O, k) + d(O, l) = \frac{24}{5} + \frac{10}{5} = \frac{34}{5} = d(P, k)$

Opgave 30:

$$\frac{|3x - 4y + 12|}{\sqrt{25}} = \frac{|4x - 3y + 6|}{\sqrt{25}}$$

$$|3x - 4y + 12| = |4x - 3y + 6|$$

$$3x - 4y + 12 = 4x - 3y + 6 \quad \vee \quad 3x - 4y + 12 = -4x + 3y - 6$$

$$-x - y = -6 \quad \vee \quad 7x - 7y = -18$$

$$m: x + y = 6 \quad n: 7x - 7y = -18$$

Opgave 31:

a. $P(0, -2)$ ligt op lijn l

$$d(P, k) = \frac{|3 \cdot 0 + -4 \cdot -2 + 12|}{\sqrt{25}} = \frac{|20|}{5} = 4$$

b. $P(x, y)$ ligt op de middenparallel m van k en l , dus $d(P, k) = d(P, l)$

$$\frac{|3x - 4y + 12|}{\sqrt{25}} = \frac{|3x - 4y - 8|}{\sqrt{25}}$$

$$|3x - 4y + 12| = |3x - 4y - 8|$$

$$3x - 4y + 12 = 3x - 4y - 8 \quad \vee \quad 3x - 4y + 12 = -3x + 4y + 8$$

$$12 = -8 \quad \vee \quad 6x - 8y = -4$$

$$\text{kan niet} \quad \vee \quad 3x - 4y = -2$$

$$\text{dus } 3x - 4y = -2$$

Opgave 32:

a. $\frac{|3x + 4y - 12|}{\sqrt{25}} = 2$

$$|3x + 4y - 12| = 10$$

$$3x + 4y - 12 = 10 \quad \vee \quad 3x + 4y - 12 = -10$$

$$3x + 4y = 22 \quad \vee \quad 3x + 4y = 2$$

b. $P(p, 0)$

$$\frac{|3p - 12|}{\sqrt{25}} = 3$$

$$|3p - 12| = 15$$

$$3p - 12 = 15 \quad \vee \quad 3p - 12 = -15$$

$$3p = 27 \quad \vee \quad 3p = -3$$

$$p = 9 \quad \vee \quad p = -1$$

$$(9, 0) \text{ en } (0, -1)$$

Opgave 33:

a. lijn AB : $\frac{x}{6} + \frac{y}{8} = 1$

$$8x + 6y = 48$$

$$d(P, C) = \frac{|8 \cdot 11 + 6 \cdot 12 - 48|}{\sqrt{100}} = 11,2$$

b. $rc_{BC} = \frac{12 - 0}{11 - 6} = \frac{12}{5}$

$$\text{lijn } BC: y - 0 = \frac{12 - 0}{11 - 6} \cdot (x - 6)$$

$$y = \frac{12}{5}(x - 6)$$

$$rc_{AD} = \frac{-1}{rc_{BC}} = \frac{-1}{\frac{12}{5}} = -\frac{5}{12}$$

lijn AD : $y = -\frac{5}{12}x + b$ door $(0,8)$

$$y = -\frac{5}{12}x + 8$$

snijpunt van AD en BC :

$$\frac{12}{5}(x-6) = -\frac{5}{12}x + 8$$

$$\frac{12}{5}x - 14\frac{2}{5} = -\frac{5}{12}x + 8$$

$$2\frac{49}{60}x = 22\frac{2}{5}$$

$$x = 7\frac{161}{169}$$

$$y = 4\frac{116}{169}$$

c. lijn AB : $8x + 6y = 48$ ofwel $4x + 3y = 24$

lijn BC : $y = \frac{12}{5}(x-6)$

$$5y = 12(x-6)$$

$$5y = 12x - 72$$

$$12x - 5y = 72$$

$$\frac{|4x + 3y - 24|}{\sqrt{25}} = \frac{|12x - 5y - 72|}{\sqrt{169}}$$

$$13 \cdot |4x + 3y - 24| = 5 \cdot |12x - 5y - 72|$$

$$52x + 39y - 312 = 60x - 25y - 360 \quad \vee \quad 52x + 39y - 312 = -60x + 25y + 360$$

$$-8x + 64y = -48 \quad \vee \quad 112x + 14y = 672$$

$$x - 8y = 6 \quad \vee \quad 8x + y = 48$$

de lijn $x - 8y = 6$ snijdt het lijnstuk AC niet

lijn AC : $y - 8 = \frac{12-8}{11-0} \cdot (x-0)$

$$y - 8 = \frac{4}{11}x$$

$$y = \frac{4}{11}x + 8$$

AC snijden met de lijn $8x + y = 48$

$$8x + \frac{4}{11}x + 8 = 48$$

$$8\frac{4}{11}x = 40$$

$$x = 4\frac{18}{23}$$

$$y = \frac{4}{11} \cdot 4\frac{18}{23} + 8 = 9\frac{17}{23}$$

$$E(4\frac{18}{23}, 9\frac{17}{23})$$