

9.7 Diagnostische toets

Opgave 1:

a. $3p - 4 = p + 2$

$$2p = 6$$

$$p = 3$$

b. $\frac{3p}{1} = \frac{3}{4}$

$$3p = \frac{3}{4}$$

$$p = \frac{1}{4}$$

c. $\frac{x}{4} + \frac{y}{5} = 3$

$$5x + 4y = 60$$

$$\frac{3p}{1} = \frac{5}{4}$$

$$3p = \frac{5}{4}$$

$$p = \frac{5}{12}$$

d. $rc_{x-as} = 0$

$$3p = 0$$

$$p = 0$$

Opgave 2:

a. $\frac{3}{p} = \frac{p}{p+6}$

$$p^2 = 3(p+6)$$

$$p^2 = 3p + 18$$

$$p^2 - 3p - 18 = 0$$

$$(p-6)(p+3) = 0$$

$$p = 6 \quad \vee \quad p = -3$$

b. $k_p : 3x + py = 6$

$$py = -3x + 6$$

$$y = -\frac{3}{p}x + \frac{6}{p}$$

$$rc_{k_p} = -\frac{3}{p}$$

$$rc_k \cdot rc_l = -1$$

$$-\frac{3}{p} \cdot -\frac{p}{p+6} = -1$$

$$\frac{3p}{p(p+6)} = -1$$

$$3p = -p(p+6)$$

$$3p = -p^2 - 6p$$

$$p^2 + 9p = 0$$

$$p(p+9) = 0$$

$$p = 0 \quad \vee \quad p = -9$$

c. snijpunt met de y-as dus $x = 0$

$$l_p : px + (p+6)y = p+2$$

$$(p+6)y = -px + p+2$$

$$y = -\frac{p}{p+6}x + \frac{p+2}{p+6}$$

$$rc_{l_p} = -\frac{p}{p+6}$$

$$\begin{cases} py = 6 \\ (p+6)y = p+2 \end{cases}$$

$$\begin{cases} y = \frac{6}{p} \\ (p+6)y = p+2 \end{cases}$$

$$(p+6) \cdot \frac{6}{p} = p+2$$

$$(p+6) \cdot 6 = p(p+2)$$

$$6p+36 = p^2+2p$$

$$p^2-4p-36 = 0$$

$$p = \frac{4 \pm \sqrt{160}}{2} = 2 \pm \sqrt{40} = 2 \pm 2\sqrt{10}$$

$$p = 2 + 2\sqrt{10} \quad \vee \quad p = 2 - 2\sqrt{10}$$

Opgave 3:

a. $r = \sqrt{2^2 + 6^2} = \sqrt{40}$

$$(x-2)^2 + (y+1)^2 = 40$$

b. k is de middelloodlijn van AB

$$\sqrt{(x-2)^2 + (y+1)^2} = \sqrt{(x-4)^2 + (y-5)^2}$$

$$(x-2)^2 + (y+1)^2 = (x-4)^2 + (y-5)^2$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 = x^2 - 8x + 16 + y^2 - 10y + 25$$

$$4x + 12y = 36$$

$$x + 3y = 9$$

l is de middelloodlijn van AC

$$\sqrt{(x-2)^2 + (y+1)^2} = \sqrt{(x-6)^2 + (y-3)^2}$$

$$(x-2)^2 + (y+1)^2 = (x-6)^2 + (y-3)^2$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 = x^2 - 12x + 36 + y^2 - 6y + 9$$

$$8x + 8y = 40$$

$$x + y = 5$$

M is het snijpunt van k en l :

$$\begin{cases} x + 3y = 9 \\ x + y = 5 \quad - \end{cases}$$

$$2y = 4$$

$$y = 2$$

$$x = 3 \text{ dus } M(3,2)$$

$$r = d(A, M) = \sqrt{1^2 + 3^2} = \sqrt{10}$$

c. M is het midden van AB

$$M(3,2)$$

$$r = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$(x-3)^2 + (y-2)^2 = 10$$

Opgave 4:

$$a. \frac{|x+2y-6|}{\sqrt{5}} = \frac{|-2x+y-10|}{\sqrt{5}}$$

$$|x+2y-6| = |-2x+y-10|$$

$$x+2y-6 = -2x+y-10 \quad \vee \quad x+2y-6 = 2x-y+10$$

$$3x+y = -4 \quad \vee \quad -x+3y = 16$$

$$b. P(0, p)$$

$$\frac{|2p-6|}{\sqrt{5}} = 2$$

$$|2p-6| = 2\sqrt{5}$$

$$2p-6 = 2\sqrt{5} \quad \vee \quad 2p-6 = -2\sqrt{5}$$

$$2p = 6+2\sqrt{5} \quad \vee \quad 2p = 6-2\sqrt{5}$$

$$p = 3+\sqrt{5} \quad \vee \quad p = 3-\sqrt{5}$$

$$P(0, 3+\sqrt{5}) \text{ of } P(0, 3-\sqrt{5})$$

Opgave 5:

a. de lijn BC is een verticale lijn, dus lijn BC is $x = 4$

$$d(A, BC) = 2$$

b. lijn AB : $rc = \frac{2-1}{4-2} = \frac{1}{2}$

$$y = \frac{1}{2}x + b \text{ door } (2,1)$$

$$1 = 1 + b$$

$$b = 0$$

$$y = \frac{1}{2}x \text{ ofwel } x - 2y = 0$$

willekeurig punt P op de x -as: $P(p, 0)$

$$d(P, AB) = \frac{|p|}{\sqrt{5}}$$

$$d(P, BC) = \frac{|p-4|}{\sqrt{1}}$$

$$\frac{|p|}{\sqrt{5}} = \frac{|p-4|}{1}$$

$$|p| = \sqrt{5} \cdot |p-4|$$

$$p = \sqrt{5} \cdot (p-4) \quad \vee \quad p = -\sqrt{5} \cdot (p-4)$$

$$p = p\sqrt{5} - 4\sqrt{5} \quad \vee \quad p = -p\sqrt{5} + 4\sqrt{5}$$

$$p - p\sqrt{5} = -4\sqrt{5} \quad \vee \quad p + p\sqrt{5} = 4\sqrt{5}$$

$$p(1-\sqrt{5}) = -4\sqrt{5} \quad \vee \quad p(1+\sqrt{5}) = 4\sqrt{5}$$

$$p = \frac{-4\sqrt{5}}{1-\sqrt{5}} = \frac{-4\sqrt{5}}{1-\sqrt{5}} \cdot \frac{1+\sqrt{5}}{1+\sqrt{5}} = \frac{-4\sqrt{5}-20}{1-5} = \sqrt{5} + 5$$

$$\vee \quad p = \frac{4\sqrt{5}}{1+\sqrt{5}} = \frac{4\sqrt{5}}{1+\sqrt{5}} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{4\sqrt{5}-20}{1-5} = -\sqrt{5} + 5$$

$$P(5 + \sqrt{5}, 0) \vee P(5 - \sqrt{5}, 0)$$

Opgave 6:

a. $k: 2x + 3y = 12$

$$\underline{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\underline{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

lijn k gaat door $(6, 0)$

$$\begin{cases} x = 6 + 3\lambda \\ y = -2\lambda \end{cases}$$

b. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$$\underline{r}_l = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \text{ dus } \underline{n}_l = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$l: 4x + 3y = c$ door $(-2, 5)$

$l: 4x + 3y = 7$

Opgave 7:

Stel $Q(p, \frac{3}{4}p)$

dan $OQ = \sqrt{p^2 + (\frac{3}{4}p)^2} = \sqrt{p^2 + \frac{9}{16}p^2} = \sqrt{\frac{25}{16}p^2} = \frac{5}{4}p$

dan $P(\frac{15}{4}p, 0)$

$M(\frac{19}{8}p, \frac{3}{8}p)$

$$rc_{OM} = \frac{\frac{3}{8}p}{\frac{19}{8}p} = \frac{3}{19}$$

lijn $OM: y = \frac{3}{19}x$ ofwel $3x - 19y = 0$

Opgave 8:

a. $x^2 + y^2 + ax - 4y + 13 = 0$

$$(x + \frac{1}{2}a)^2 - \frac{1}{4}a^2 + (y - 2)^2 - 4 + 13 = 0$$

$$(x + \frac{1}{2}a)^2 + (y - 2)^2 = \frac{1}{4}a^2 - 9$$

$$\frac{1}{4}a^2 - 9 \geq 0$$

$$\frac{1}{4}a^2 \geq 9$$

$$a^2 \geq 36$$

$$a \leq -6 \vee a \geq 6$$

b. $M(-\frac{1}{2}a, 2)$

als M op l ligt dan geldt: $2 \cdot -\frac{1}{2}a + 3 \cdot 2 = 5$

$$-a + 6 = 5$$

$$-a = -1$$

$$a = 1$$

Maar voor $a = 1$ heb je geen cirkel, dus M ligt niet op l .

Opgave 9:

a. $k: -x - 3y = 10$

dus $x + 3y = 10$

b. $l: 3x - y + c = 0$

$$d(M, l) = r \text{ dus } \frac{|0 - 0 + c|}{\sqrt{3^2 + (-1)^2}} = \sqrt{10}$$

$$|c| = 10$$

$$c = 10 \quad \vee \quad c = -10$$

$$l_1: 3x - y + 10 = 0 \text{ en } l_2: 3x - y - 10 = 0$$

c. lijn l door $(4, 2)$ dus $y - 2 = a(x - 4)$

$$ax - y + 2 - 4a = 0$$

$$d(M, l) = r \text{ dus } \frac{|0 - 0 + 2 - 4a|}{\sqrt{a^2 + 1}} = \sqrt{10}$$

$$|2 - 4a| = \sqrt{10a^2 + 10}$$

$$16a^2 - 16a + 4 = 10a^2 + 10$$

$$6a^2 - 16a - 6 = 0$$

$$a = \frac{16 \pm \sqrt{400}}{12} = \frac{16 \pm 20}{12}$$

$$a = 3 \quad \vee \quad a = -\frac{1}{3}$$

$$l_1: y - 2 = 3(x - 4) \text{ en } l_2: y - 2 = -\frac{1}{3}(x - 4)$$

d. $m: 3x + y - 5 = 0$

$$l \perp m \text{ dus } l: x - 3y + c = 0$$

$$d(M, l) = r \text{ dus } \frac{|0 - 0 + c|}{\sqrt{1^2 + (-3)^2}} = \sqrt{10}$$

$$|c| = 10$$

$$l_1: x - 3y + 10 = 0 \text{ en } l_2: x - 3y - 10 = 0$$

Opgave 10:

a. $x^2 + y^2 - 6x - 4y = 0$

$$(x - 3)^2 - 9 + (y - 2)^2 - 4 = 0$$

$$(x - 3)^2 + (y - 2)^2 = 13$$

poellijn p ten opzichte van $A(-3, 4)$ is:

$$(-3 - 3)(x - 3) + (4 - 2)(y - 2) = 13$$

$$-6(x - 3) + 2(y - 2) = 13$$

$$-6x + 18 + 2y - 4 = 13$$

$$-6x + 2y = -1$$

b. poellijn p ten opzichte van $B(2, -3)$ is:

$$(2 - 3)(x - 3) + (-3 - 2)(y - 2) = 13$$

$$-(x - 3) - 5(y - 2) = 13$$

$$-x + 3 - 5y + 10 = 13$$

$$x + 5y = 0$$

$$x = -5y$$

p snijden met c geeft:

$$25y^2 + y^2 + 30y - 4y = 0$$

$$26y^2 + 26y = 0$$

$$26y(y+1) = 0$$

$$y = 0 \quad \vee \quad y = -1$$

raakpunten $(0,0)$ en $(5,-1)$

raaklijn in $(0,0)$ is: $(0-3)(x-3) + (0-2)(y-2) = 13$

$$-3x + 9 - 2y + 4 = 13$$

$$3x + 2y = 0$$

raaklijn in $(5,-1)$ is: $(5-3)(x-3) + (-1-2)(y-2) = 13$

$$2(x-3) - 3(y-2) = 13$$

$$2x - 6 - 3y + 6 = 13$$

$$2x - 3y = 13$$

Opgave 11:

a. $r^2 = (-2)^2 + (-2)^2 - 6 \cdot -2 + 8 \cdot -2 + 5 = 9$

$$c_2: (x+2)^2 + (y+2)^2 = 9$$

b. $(3+2\lambda)^2 + (-4+\lambda)^2 - 6(3+2\lambda) + 8(-4+\lambda) + 5 = 5$

$$4\lambda^2 + 12\lambda + 9 + \lambda^2 - 8\lambda + 16 - 18 - 12\lambda - 32 + 8\lambda + 5 = 5$$

$$5\lambda^2 = 25$$

$$\lambda^2 = 5$$

$$\lambda = \sqrt{5} \quad \vee \quad \lambda = -\sqrt{5}$$

$$(3+2\sqrt{5}, -4+\sqrt{5}) \text{ en } (3-2\sqrt{5}, -4-\sqrt{5})$$

Opgave 12:

$$c_1: x^2 + y^2 - 4x - 2y = 0$$

$$c_2: (x-4)^2 + (y+1)^2 = 1$$

$$x^2 - 8x + 16 + y^2 + 2y + 1 = 1$$

$$x^2 - 8x + y^2 + 2y + 16 = 0$$

machtlijn m ten opzichte van c_1 en c_2 is:

$$(-4 - -8)x + (-2 - 2)y - 16 = 0$$

$$4x - 4y = 16$$

$$x - y = 4 \text{ dus } x = y + 4$$

m snijden met c_1 geeft:

$$(y+4)^2 + y^2 - 4(y+4) - 2y = 0$$

$$y^2 + 8y + 16 + y^2 - 4y - 16 - 2y = 0$$

$$2y^2 + 2y = 0$$

$$2y(y+1) = 0$$

$$y = 0 \quad \vee \quad y = -1$$

$x = 4 \vee x = 3$
dus $(4,0)$ en $(3,-1)$

Opgave 13:

stel het raakpunt P is het punt $P(x, y)$

dan is: $rc_{OP} = \frac{y}{x}$

$$rc_{PM} = \frac{0-y}{10-x} = \frac{y}{x-10}$$

$PM \perp OM$ dus $rc_{PM} \cdot rc_{OM} = -1$

$$\frac{y}{x-10} \cdot \frac{y}{x} = -1$$

$$\frac{y^2}{x^2 - 10x} = -1$$

$$y^2 = -1(x^2 - 10x)$$

$$y^2 = -x^2 + 10x$$

$$x^2 - 10x + y^2 = 0$$

$$(x-5)^2 - 25 + y^2 = 0$$

$$(x-5)^2 + y^2 = 25$$

dus alle punten P liggen op de cirkel met middelpunt $(5,0)$ en straal 5