14.2 Parametervoorstellingen

Opgave 17:

a. $y = 2 - \lambda$

$\lambda = 2 - y$

$x = 3 + 4(2 - y)$

$x = 3 + 8 - 4y$

$x + 4y = 11$

b. $(15,-1)$

c. $x = 3 + 4\lambda = 0$

$4\lambda = -3$

$\lambda = -\frac{3}{4}$

Opgave 18:

a. $x^2 + y^2 - 4x - 2y - 8 = 0$

$(x-2)^2 - 4 + (y-1)^2 - 1 - 8 = 0$

$(x-2)^2 + (y-1)^2 = 13$ dus $M(2,1)$ en $r = \sqrt{13}$

l: $y = a(x+6)$

$y = ax + 6a$

$ax - y + 6a = 0$

$d(M,l) = r$ dus $\frac{|2a - 1 + 6a|}{\sqrt{a^2+1}} = \sqrt{13}$

$\frac{|8a-1|}{\sqrt{a^2+1}} = \sqrt{13}$

$|8a-1| = \sqrt{13a^2+13}$

$(8a-1)^2 = 13a^2 + 13$

$64a^2 - 16a + 1 = 13a^2 + 13$

$51a^2 - 16a - 12 = 0$

$a = \frac{16 \pm \sqrt{2704}}{102} = \frac{16 \pm 52}{102}$

$a = -\frac{6}{17} \quad \vee \quad a = \frac{2}{3}$

l_1: $y = -\frac{6}{17}(x+6)$ en l_2: $y = \frac{2}{3}(x+6)$

b. de poollijn van $(-6,0)$ ten opzichte van de cirkel is:

$(-6-2)(x-2) + (0-1)(y-1) = 13$

$-8(x-2) - (y-1) = 13$

$-8x + 16 - y + 1 = 13$

$-y = 8x - 4$

$y = -8x + 4$

de poollijn snijden met de cirkel geeft:

$(x-2)^2 + (-8x + 4 - 1)^2 = 13$

$x^2 - 4x + 4 + 64x^2 - 48x + 9 = 13$

$65x^2 - 52x = 0$

$x(65x-52)=0$

$x=0 \quad \vee \quad 65x=52$

$x=0 \quad \vee \quad x=\frac{4}{5}$

$y=4 \qquad y=-2\frac{2}{5}$

raaklijn in $(0,4)$ is: $(0-2)(x-2)+(4-1)(y-1)=13$

$-2(x-2)+3(y-1)=13$

$-2x+4+3y-3=13$

$-2x+3y=12$

raaklijn in $(\frac{4}{5},-2\frac{2}{5})$ is: $(\frac{4}{5}-2)(x-2)+(-2\frac{2}{5}-1)(y-1)=13$

$-1\frac{1}{5}(x-2)-3\frac{2}{5}(y-1)=13$

$-6(x-2)-17(y-1)=65$

$-6x+12-17y+17=65$

$-6x-17y=36$

$6x+17y=-36$

Opgave 19:

a. $x=-13+\lambda$

$y=a\lambda$

b. $9(-13+\lambda)^2+25(a\lambda)^2-36(-13+\lambda)-150a\lambda+36=0$

$9(169-26\lambda+\lambda^2+25a^2\lambda^2+468-36\lambda-150a\lambda+36=0$

$1521-234\lambda+9\lambda^2+25a^2\lambda^2+468-36\lambda-150a\lambda+36=0$

$(25a^2+9)\lambda^2+(-150a-270)\lambda+2025=0$

$D=(-150a-270)^2-4(25a^2+9)\cdot 2025=0$

$22500a^2+81000a+72900-202500a^2-72900=0$

$-180000a^2+81000a=0$

$a(-180000a+81000)=0$

$a=0 \quad \vee \quad -180000a=-81000$

$a=0 \quad \vee \quad a=\frac{9}{20}$

$\bar{r}_l=\begin{pmatrix}1\\0\end{pmatrix}$ dus $\bar{n}_l=\begin{pmatrix}0\\1\end{pmatrix}$ dus $l_1: \quad y=0$

$\bar{r}_l=\begin{pmatrix}1\\\frac{9}{20}\end{pmatrix}=\begin{pmatrix}20\\9\end{pmatrix}$ dus $\bar{n}_l=\begin{pmatrix}9\\-20\end{pmatrix}$ dus $l_2: \quad 9x-20y=-117$

Opgave 20:

lijn l: $x=\lambda$ en $y=-8\frac{1}{2}+a\lambda$

$9\lambda^2-16(-8\frac{1}{2}+a\lambda)^2-36\lambda+96(-8\frac{1}{2}+a\lambda)-368=0$

$9\lambda^2-16(72\frac{1}{4}-17a\lambda+\lambda^2)-36\lambda-816+96a\lambda-368=0$

$9\lambda^2-1156+272a\lambda-16a^2\lambda^2-36\lambda-816+96a\lambda-368=0$

$(9-16a^2)\lambda^2+(368a-36)\lambda-2340=0$

$D=(368a-36)^2-4(9-16a^2)\cdot -2340=0$

$135424a^2-26496a+1296+84240-149760a^2=0$

$-14336a^2 - 26496a + 85536 = 0$

$a = \dfrac{26496 \pm \sqrt{5607014400}}{-28672} = \dfrac{26496 \pm 74880}{-28672}$

$a = -\frac{99}{28} \quad \vee \quad a = \frac{27}{16}$

$\bar{r}_l = \begin{pmatrix} 1 \\ -\frac{99}{28} \end{pmatrix} = \begin{pmatrix} 28 \\ -99 \end{pmatrix}$ dus $\bar{n}_l = \begin{pmatrix} 99 \\ 28 \end{pmatrix}$ dus $l_1: \quad 99x + 28y = -238$

$\bar{r}_l = \begin{pmatrix} 1 \\ \frac{27}{16} \end{pmatrix} = \begin{pmatrix} 16 \\ 27 \end{pmatrix}$ dus $\bar{n}_l = \begin{pmatrix} 27 \\ -16 \end{pmatrix}$ dus $l_2: \quad 27x - 16y = 136$

Opgave 21:

a. $x^2 + y^2 = (\cos\varphi)^2 + (\sin\varphi)^2 = \cos^2\varphi + \sin^2\varphi = 1$

b. de eenheidscirkel

Opgave 22:

a. $x_Q = \dfrac{-4 + 3\cos\varphi}{2} = -2 + 1\frac{1}{2}\cos\varphi$

$y_Q = \dfrac{2 + 3\sin\varphi + 6}{2} = 4 + 1\frac{1}{2}\sin\varphi$

R ligt op de cirkel: $(x+2)^2 + (y-4)^2 = 2\frac{1}{4}$

b. $S(a,0)$

$x_S = \dfrac{-4 + 3\cos\varphi + a}{2} = \frac{1}{2}a - 2 + 1\frac{1}{2}\cos\varphi$

$\frac{1}{2}a - 2 = 5$

$\frac{1}{2}a = 7$

$a = 14$ dus $S(14,0)$

Opgave 23:

a. $x = 3\cos\varphi$

$y = 2\sin\varphi$

b. $\cos\varphi = \frac{1}{3}x$

$\sin\varphi = \frac{1}{2}y$

$\cos^2\varphi + \sin^2\varphi = 1$

$(\frac{1}{3}x)^2 + (\frac{1}{2}y)^2 = 1$

$\frac{1}{9}x^2 + \frac{1}{4}y^2 = 1$

$4x^2 + 9y^2 = 36$

Opgave 24:

a. $4\cos\varphi = x - 3$

$2\sin\varphi = y - 1$

$4\sin\varphi = 2y - 2$

$16\cos^2\varphi + 16\sin^2\varphi = 16$

$(4\cos\varphi)^2 + (4\sin\varphi)^2 = 16$

$(x-3)^2+(2y-2)^2=16$

b. $(x-3)^2+(2(y-1))^2=16$

$(x-3)^2+4(y-1)^2=16$

$\frac{(x-3)^2}{16}+\frac{(y-1)^2}{4}=1$

middelpunt $M(3,1)$ en toppen $(-1,1)$, $(7,1)$, $(3,-1)$, $(3,3)$

$c^2=a^2-b^2=16-4=12$ dus brandpunten $F_1(3-2\sqrt{3},1)$ en $F_2(3+2\sqrt{3},1)$

Opgave 25:

a. het middelpunt is $(-4,2)$

$a=3 \quad b=2$

$\begin{cases} x=-4+3\cos\varphi \\ y=2+2\sin\varphi \end{cases}$

b. $x_Q=\frac{1}{2}(-4+3\cos\varphi-2)=-3+1\frac{1}{2}\cos\varphi$

$y_Q=\frac{1}{2}(2+2\sin\varphi)=1+\sin\varphi$

$1\frac{1}{2}\cos\varphi=x+3$

$\cos\varphi=\frac{2}{3}(x+3)$

$\sin\varphi=y-1$

$\cos^2\varphi+\sin^2\varphi=1$

$(\frac{2}{3}(x+3))^2+(y-1)^2=1$

$\frac{4}{9}(x+3)^2+(y-1)^2=1$

$4(x+3)^2+9(y-1)^2=9$

c. $a=3$ en $b=2$ dus je moet vermenigvuldigen met $1\frac{1}{2}$

$\begin{cases} x=-4+3\cos\varphi \\ y=3+3\sin\varphi \end{cases}$

$c_1: \quad (x+4)^2+(y-3)^2=9$

d. je moet vermenigvuldigen met $\frac{2}{3}$

$\begin{cases} x=-2\frac{2}{3}+2\cos\varphi \\ y=2+2\sin\varphi \end{cases}$

$c_2: \quad (x+2\frac{2}{3})^2+(y-2)^2=4$

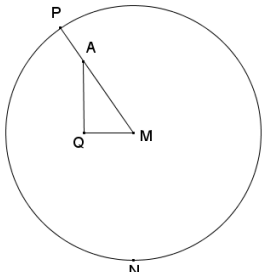
Opgave 26:

$omtrek\ cirkel=2\pi r$

je hebt het $\frac{\varphi}{2\pi}$ e deel van de cirkel

$boog\ AB=\frac{\varphi}{2\pi}\cdot 2\pi r=r\cdot\varphi$

Opgave 27:

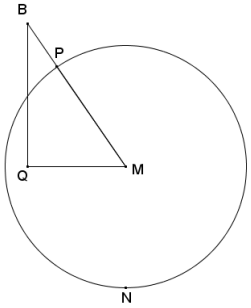


a. $AQ = AM \cdot \sin(\varphi - \frac{1}{2}\pi) = a \cdot \sin(\varphi - \frac{1}{2}\pi) = -a \cdot \cos\varphi$

$y_A = MN + AQ = r + AQ = r - a \cdot \cos\varphi$

$QM = AM \cdot \cos(\varphi - \frac{1}{2}\pi) = a \cdot \sin\varphi$

$x_A = ON - QM = r \cdot \varphi - a \cdot \sin\varphi$



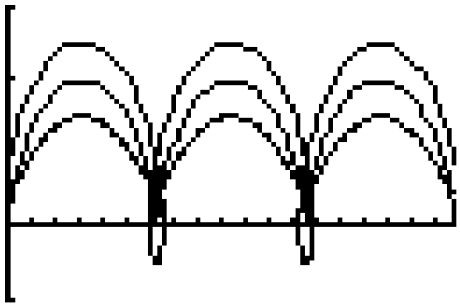
b. $BQ = BM \cdot \sin(\varphi - \frac{1}{2}\pi) = -b \cdot \cos\varphi$

$y_B = MN + BQ = r - b \cdot \cos\varphi$

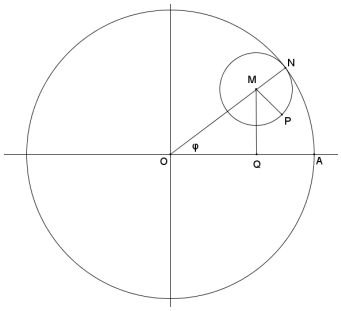
$QM = BM \cdot \cos(\varphi - \frac{1}{2}\pi) = b \cdot \sin\varphi$

$x_B = ON - QM = r\varphi - b \cdot \sin\varphi$

c.



Opgave 28:



a. R is de straal van de grote cirkel en r is de straal van de kleine cirkel

$\cos\varphi = \dfrac{OQ}{OM}$

$OQ = OM \cdot \cos\varphi = (ON - OM) \cdot \cos\varphi = (R - r)\cos\varphi$

dus $x_M = (R - r)\cos\varphi$

$\sin\varphi = \dfrac{QM}{OM}$

$QM = OM \cdot \sin\varphi = (ON - MN)\sin\varphi = (R - r)\sin\varphi$

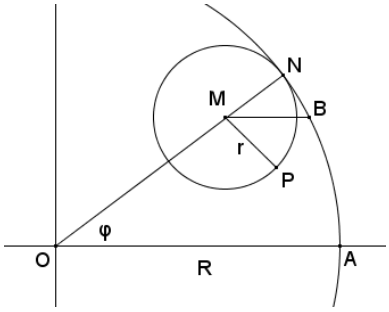
dus $y_M = (R - r)\sin\varphi$

b. $boog\ AN = R\varphi$ en $boog\ NP = r \cdot \angle NMP$

$boog\ AN = noog\ NP$

$R\varphi = r \cdot \angle NMP$

$\angle NMP = \dfrac{R}{r} \cdot \varphi$



c. MB is evenwijdig met OA dus $\angle BMN = \varphi$

$\angle BMP = \angle NMP - \angle BMN = \dfrac{R}{r} \cdot \varphi - \varphi$

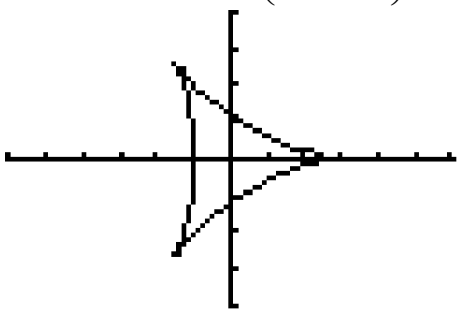
punt P draait in negatieve richting rond M dus de draaihoek van MP is: $-\left(\dfrac{R}{r} \cdot \varphi - \varphi\right) = \varphi - \dfrac{R}{r} \cdot \varphi$

d. de draaihoek van OM is φ en dr draaihoek van MP is $\varphi - \frac{R}{r} \cdot \varphi$

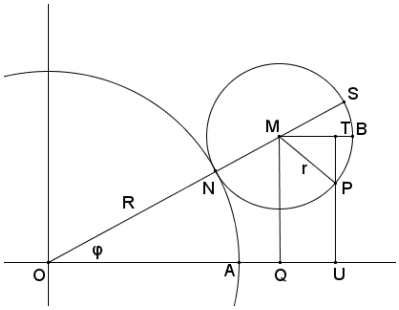
$$x_P = x_M + MP \cdot \cos\left(\varphi - \frac{R}{r} \cdot \varphi\right) = (R-r)\cos\varphi + r\cos\left(\varphi - \frac{R}{r} \cdot \varphi\right)$$

$$y_P = y_M + MP \cdot \sin\left(\varphi - \frac{R}{r} \cdot \varphi\right) = (R-r)\sin\varphi + r\sin\left(\varphi - \frac{R}{r} \cdot \varphi\right)$$

e.



Opgave 29:



$$\cos\angle MOQ = \frac{OQ}{OM}$$

$$OQ = OM \cdot \cos\angle MOQ = (ON + NM)\cos\varphi$$

$$x_M = (R+r)\cos\varphi$$

$$\sin\angle MOQ = \frac{MQ}{OQ}$$

$$MQ = OQ \cdot \sin\angle MOQ = (ON + NM)\sin\varphi$$

$$y_M = (R+r)\sin\varphi$$

$boog\ AN = R\varphi$ en $boog\ NP = r \cdot \angle NMP$

$boog\ AN = boog\ NP$

$$R\varphi = r \cdot \angle NMP$$

$$\angle NMP = \frac{R}{r} \cdot \varphi$$

MB is evenwijdig met de x-as dus $\angle BMS = \angle UOS = \varphi$

$$\angle BMP = \pi - \angle BMS - \angle NMP = \pi - \varphi - \frac{R}{r} \cdot \varphi$$

$$\cos\angle PMT = \frac{MT}{MP}$$

$$MT = MP \cdot \cos\angle PMT = MP \cdot \cos\angle BMP = r\cos\left(\pi - \varphi - \frac{R}{r} \cdot \varphi\right)$$

$$= r\cos\left(\pi - \left(\varphi + \frac{R}{r} \cdot \varphi\right)\right) = -r\cos\left(\varphi + \frac{R}{r} \cdot \varphi\right)$$

$$\sin\angle PMT = \frac{PT}{MP}$$

$$PT = MP \cdot \sin\angle PMT = MP \cdot \sin\angle BMP = r\sin\left(\pi - \varphi - \frac{R}{r} \cdot \varphi\right)$$

$$= r\sin\left(\pi - \left(\varphi + \frac{R}{r}\cdot\varphi\right)\right) = r\sin\left(\varphi + \frac{R}{r}\cdot\varphi\right)$$

$$x_P = OQ + QU = x_M + MT = (R+r)\cos\varphi - r\cos\left(\varphi + \frac{R}{r}\cdot\varphi\right)$$

$$y_P = UT - PT = y_M - PT = (R+r)\sin\varphi - r\sin\left(\varphi + \frac{R}{r}\cdot\varphi\right)$$