14.3 Het differentiaalquotiënt bij krommen

Opgave 30:

a. $\frac{dx}{dt} = x'(t) = 2t - 4$

$\frac{dy}{dt} = y'(t) = 2$

b. punt T is het punt op de parabool waarvan de x-coördinaat minimaal is, dus $x'(t) = 0$, dus $\frac{dx}{dt} = 0$

c. $2t - 4 = 0$

$2t = 4$

$t = 2$

$T(-4,-2)$

Opgave 31:

a. evenwijdig aan de x-as dus: $\frac{dy}{dt} = 0$

$\frac{dy}{dt} = 3t^2 - 3 = 0$

$3t^2 = 3$

$t^2 = 1$

$t = 1 \quad \vee \quad t = -1$

$(-3,-2)$ en $(-3,2)$

evenwijdig aan de y-as dus: $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = 2t = 0$

$t = 0$

$(-4,0)$

b. $x = -1$ dus $t^2 - 4 = -1$

$t^2 = 3$

$t = \sqrt{3} \quad \vee \quad t = -\sqrt{3}$

$y = 0$ dus $t^3 - 3t = 0$

$t(t^2 - 3) = 0$

$t = 0 \quad \vee \quad t^2 = 3$

$t = 0 \quad \vee \quad t = \sqrt{3} \quad \vee \quad t = -\sqrt{3}$

dus K is voor $t = -\sqrt{3}$ en voor $t = \sqrt{3}$ in het punt $(-1,0)$

$\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$

voor $t = -\sqrt{3}$ geldt: $\frac{dy}{dx} = \frac{6}{-2\sqrt{3}} = -\sqrt{3}$

$y = -\sqrt{3} \cdot x + b$ door $(-1,0)$

$0 = \sqrt{3} + b$

$b = -\sqrt{3}$

$y = -\sqrt{3} \cdot x - \sqrt{3}$

voor $t = \sqrt{3}$ geldt: $\frac{dy}{dx} = \frac{6}{2\sqrt{3}} = \sqrt{3}$

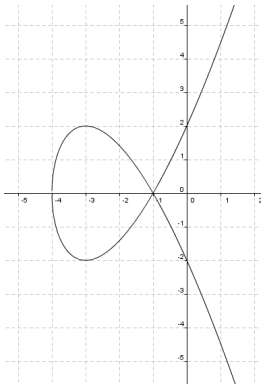
$y = \sqrt{3} \cdot x + b$ door $(-1,0)$

$0 = -\sqrt{3} + b$

$b = \sqrt{3}$

$y = \sqrt{3} \cdot x + \sqrt{3}$

de grafiek snijdt zichzelf niet loodrecht, want $\sqrt{3} \cdot -\sqrt{3} \neq -1$

c. 

Opgave 32:

a. snijpunten x-as: $y = 4\sin t = 0$

$t = 0 \quad \vee \quad t = \pi \quad \vee \quad t = 2\pi$

(1,0) en (–3,0)

snijpunten y-as: $x = 2\cos t - 1 = 0$

$2\cos t = 1$

$\cos t = \frac{1}{2}$

$t = \frac{1}{3}\pi \quad \vee \quad t = \frac{5}{3}\pi$

$(0,2\sqrt{3})$ en $(0,-2\sqrt{3})$

b. $-1 \leq \cos t \leq 1$

$-2 \leq 2\cos t \leq 2$

$-3 \leq 2\cos t - 1 \leq 1$ dus $-3 \leq x \leq 1$

$-1 \leq \sin t \leq 1$

$-4 \leq 4\sin t \leq 4$ dus $-4 \leq y \leq 4$

c. evenwijdig x-as: $\frac{dy}{dt} = 4\cos t = 0$

$t = \frac{1}{2}\pi \quad \vee \quad t = 1\frac{1}{2}\pi$

(–1,4) en (–1,–4)

evenwijdig y-as: $\frac{dx}{dt} = -2\sin t = 0$

$t = 0 \quad \vee \quad t = \pi \quad \vee \quad t = 2\pi$

(1,0) en (–3,0)

d. $\frac{dy}{dx} = \frac{4\cos t}{-2\sin t} = 2$

$4\cos t = -4\sin t$

$\cos t = -\sin t$

$\cos t = \cos(t + \frac{1}{2}\pi)$

$t = t + \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad t = -t - \frac{1}{2}\pi + k \cdot 2\pi$

k.n. $2t = -\frac{1}{2}\pi + k \cdot 2\pi$

$t = -\frac{1}{4}\pi + k \cdot \pi$

$t = \frac{3}{4}\pi \quad \vee \quad t = 1\frac{3}{4}\pi$

dus $(-\sqrt{2}-1, 2\sqrt{2})$ en $(\sqrt{2}-1, -2\sqrt{2})$

e. $2\cos t = x+1$
$4\cos t = 2x+2$
$16\cos^2 t + 16\sin^2 t = 16$
$(4\cos t)^2 + (4\sin t)^2 = 16$
$(2x+2)^2 + y^2 = 16$
$4(x+1)^2 + y^2 = 16$

Opgave 33:

a. $t > 0$
$\frac{dx}{dt} = 2t - 4 = 0$
$2t = 4$
$t = 2$
$x = -4$ dus $x \geq -4$
$\frac{dy}{dt} = 1 \cdot \ln t + t \cdot \frac{1}{t} = \ln t + 1 = 0$
$\ln t = -1$
$t = e^{-1} = \frac{1}{e}$
$y = -\frac{1}{e}$ dus $y \geq -\frac{1}{e}$

b. evenwijdig x-as: $t = \frac{1}{e}$ dus $(\frac{1}{e^2} - \frac{4}{e}, -\frac{1}{e})$
evenwijdig y-as: $t = 2$ dus $(-4, 2\ln 2)$

c. $y = t \ln t = 0$
$t = 0 \quad \vee \quad \ln t = 0$
k.n. $t = 1$
$(-3,0)$
$\frac{dy}{dx} = -\frac{1}{2}$
$y = -\frac{1}{2}x + b$ door $(-3,0)$
$0 = 1\frac{1}{2} + b$
$b = -1\frac{1}{2}$
$y = -\frac{1}{2}x - 1\frac{1}{2}$

Opgave 34:

a. evenwijdig x-as: $\frac{dy}{dt} = \frac{1}{2}t - 1$
$-\frac{1}{2}t = -1$
$t = 2$
$(0,-1)$
evenwijdig y-as: $\frac{dx}{dt} = \frac{1}{2}t = 0$
$t = 0$
$(-1,0)$

b. $y = x$ dus $\frac{1}{4}t^2 - 1 = \frac{1}{4}t^2 - t$
$t = 1$
$(-\frac{3}{4}, -\frac{3}{4})$

voor $t=1$ is $\frac{dy}{dx}=\frac{-\frac{1}{2}}{\frac{1}{2}}=-1$

$y=-x+b$ door $(-\frac{3}{4},-\frac{3}{4})$

$-\frac{3}{4}=\frac{3}{4}+b$

$b=-1\frac{1}{2}$

$y=-x-1\frac{1}{2}$

c. snijpunt x-as: $\frac{1}{4}t^2-t=0$

$t(\frac{1}{4}t-1)=0$

$t=0 \quad \vee \quad \frac{1}{4}t=1$

$t=0 \quad \vee \quad t=4$

$(-1,0)$ en $(3,0)$

dus $B(3,0)$

snijpunt y-as: $\frac{1}{4}t^2-1=0$

$\frac{1}{4}t^2=1$

$t^2=4$

$t=2 \quad \vee \quad t=-2$

$(0,3)$ en $(0,-1)$

dus $C(0,3)$

in $(3,0)$ geldt: $\frac{dy}{dx}=\frac{1}{2}$ dus $y=\frac{1}{2}x+b$ door $(3,0)$

$0=1\frac{1}{2}+b$

$b=-1\frac{1}{2}$

$y=\frac{1}{2}x-1\frac{1}{2}$

in $(0,3)$ geldt: $\frac{dy}{dx}=2$ dus $y=2x+b$ door $(0,3)$

$y=2x+3$

$\frac{1}{2}x-1\frac{1}{2}=2x+3$

$-1\frac{1}{2}x=4\frac{1}{2}$

$x=-3$

$y=-3$

$D(-3,-3)$

d. $\frac{1}{4}t^2=x+1$

$y=x+1-t$ dus $t=x-y+1$

$x=\frac{1}{4}t^2-1$

$4x=t^2-4$

$4x=(x-y+1)^2-4$

Opgave 35:

a. de kettingregel

b. $(f(t))^2+4\cdot(g(t))^2=1$

$2f(t)\cdot f'(t)+8g(t)\cdot g'(t)=0$

$f(t)\cdot f'(t)+4g(t)\cdot g'(t)=0$

$x \cdot \frac{dx}{dt} + 4y \cdot \frac{dy}{dt} = 0$

c. $4y \cdot \frac{dy}{dt} = -x \cdot \frac{dx}{dt}$

$\frac{dy}{dx} = -\frac{x}{4y}$

Opgave 36:

a. $\frac{dy}{dx} = \frac{x^2 + 2y}{-2x + 2y} = 0$

$x^2 + 2y = 0$

$2y = -x^2$

$y = -\frac{1}{2}x^2$

$x^3 + 6x \cdot -\frac{1}{2}x^2 - 3(-\frac{1}{2}x^2)^2 = 0$

$x^3 - 3x^3 - \frac{3}{4}x^4 = 0$

$-2x^3 - \frac{3}{4}x^4 = 0$

$x^3(-2 - \frac{3}{4}x) = 0$

$x = 0 \quad \vee \quad -\frac{3}{4}x = 2$

k.n. $x = -\frac{8}{3}$

$y = -\frac{32}{9}$

$B(-2\frac{2}{3}, -3\frac{5}{9})$

b.