

## 15.4 Differentiaalvergelijkingen van de eerste orde

### Opgave 43:

a.  $y = at^2 + bt + c$

differentiëren geeft:  $\frac{dy}{dt} = 2at + b$

invullen geeft:  $\frac{dy}{dt} = 2(at^2 + bt + c) - 4t^2 + 4t$   
 $= 2at^2 + 2bt + 2c - 4t^2 + 4t$   
 $= (2a - 4)t^2 + (2b + 4)t + 2c$

$$\begin{cases} 2a - 4 = 0 \\ 2a = 2b + 4 \\ b = 2c \end{cases}$$

$$2a = 4$$

$$a = 2$$

$$4 = 2b + 4$$

$$b = 0$$

$$c = 0$$

$$y = 2t^2$$

b.  $\frac{dy}{dt} = 2y$

$$y = c \cdot e^{2t}$$

c.  $y = c \cdot e^{2t} + 2t^2$

differentiëren geeft:  $\frac{dy}{dt} = 2c \cdot e^{2t} + 4t$

invullen geeft:  $\frac{dy}{dt} = 2(c \cdot e^{2t} + 2t^2) - 4t^2 + 4t$   
 $= 2c \cdot e^{2t} + 4t^2 - 4t^2 + 4t$   
 $= 2c \cdot e^{2t} + 4t$  dus klopt

### Opgave 44:

a.  $y = a(t) + p(t)$

differentiëren geeft:  $\frac{dy}{dt} = a'(t) + p'(t) = f(t) \cdot dt + f(t) \cdot p(t) + g(t)$

invullen geeft:  $\frac{dy}{dt} = f(t) \cdot (a(t) + p(t)) + g(t) = f(t) \cdot a(t) + f(t) \cdot p(t) + g(t)$  klopt

b.  $\frac{dy}{dt} = f(t) \cdot y$

$$a(t) = c \cdot e^{F(t)}$$

differentiëren geeft:  $\frac{dy}{dt} = f(t) \cdot c \cdot e^{F(t)}$

invullen geeft:  $\frac{dy}{dt} = f(t) \cdot c \cdot e^{F(t)}$  dus klopt

c.  $y = a(t) + p(t) = c \cdot e^{F(t)} + p(t)$

$$c \cdot e^{F(t)} = y - p(t)$$

$$c = \frac{y - p(t)}{e^{F(t)}}$$

voor elk punt  $(t, y)$  bestaat er precies één waarde van  $c$  dus door elk punt  $(t, y)$  gaat er precies één oplossingskromme.

**Opgave 45:**

a.  $\frac{dy}{dt} = -\frac{1}{2}y + 2t - 4$

$$\frac{dy}{dt} = -\frac{1}{2}y \text{ geeft } y = c \cdot e^{-\frac{1}{2}t}$$

stel  $y = at + b$

differentiëren geeft:  $\frac{dy}{dt} = a$

$$\begin{aligned} \text{invullen geeft: } \frac{dy}{dt} &= -\frac{1}{2}(at + b) + 2t - 4 \\ &= -\frac{1}{2}at - \frac{1}{2}b + 2t - 4 \\ &= \left(-\frac{1}{2}a + 2\right)t - \frac{1}{2}b - 4 \end{aligned}$$

$$\begin{cases} -\frac{1}{2}a + 2 = 0 \\ -\frac{1}{2}b - 4 = a \end{cases}$$

$$-\frac{1}{2}a = -2$$

$$a = 4$$

$$-\frac{1}{2}b - 4 = 4$$

$$-\frac{1}{2}b = 8$$

$$b = -16$$

$y = 4t - 16$  is een particuliere oplossing

dus  $y = c \cdot e^{-\frac{1}{2}t} + 4t - 16$

b.  $\frac{dy}{dt} = 6 - 2y$

$y = 3$  is een particuliere oplossing

$$\frac{dy}{dt} = -2y \text{ geeft } y = c \cdot e^{-2t}$$

dus  $y = c \cdot e^{-2t} + 3$

c.  $\frac{dy}{dt} = 2y - 4t + 6$

$$\frac{dy}{dt} = 2y \text{ geeft } y = c \cdot e^{2t}$$

stel  $y = at + b$

differentiëren geeft:  $\frac{dy}{dt} = a$

$$\begin{aligned} \text{invullen geeft: } \frac{dy}{dt} &= 2(at + b) - 4t + 6 \\ &= 2at + 2b - 4t + 6 \\ &= (2a - 4)t + 2b + 6 \end{aligned}$$

$$\begin{cases} 2a - 4 = 0 \\ 2b + 6 = a \end{cases}$$

$$2a = 4$$

$$a = 2$$

$$2b + 6 = 2$$

$$2b = -4$$

$$b = -2$$

$y = 2t - 2$  is een particuliere oplossing

dus  $y = c \cdot e^{2t} + 2t - 2$

d.  $\frac{dT}{dt} = -\frac{1}{10}(T - 21)$

$T = 21$  is een particuliere oplossing

$$\frac{dT}{dt} = -\frac{1}{10}T \text{ geeft } T = c \cdot e^{-\frac{1}{10}t}$$

dus  $T = c \cdot e^{-\frac{1}{10}t} + 21$

**Opgave 46:**

a.  $\frac{dy}{dt} = 2y + \cos t$

$\frac{dy}{dt} = 2y$  geeft  $y = c \cdot e^{2t}$

stel  $y = A \sin t + B \cos t$

differentiëren geeft:  $\frac{dy}{dt} = A \cos t - B \sin t$

invullen geeft:  $\frac{dy}{dt} = 2(A \sin t + B \cos t) + \cos t$   
 $= 2A \sin t + 2B \cos t + \cos t$   
 $= 2A \sin t + (2B + 1) \cos t$

$$\begin{cases} 2A = -B \\ 2B + 1 = A \end{cases}$$

$$2(2B + 1) = -B$$

$$4B + 2 = -B$$

$$5B = -2$$

$$B = -\frac{2}{5}$$

$$A = \frac{1}{5}$$

$y = \frac{1}{5} \sin t - \frac{2}{5} \cos t$  is een particuliere oplossing

dus  $y = c \cdot e^{2t} + \frac{1}{5} \sin t - \frac{2}{5} \cos t$

b.  $\frac{dy}{dt} = -3y - \sin t$

$\frac{dy}{dt} = -3y$  geeft  $y = c \cdot e^{-3t}$

stel  $y = A \sin t + B \cos t$

differentiëren geeft:  $\frac{dy}{dt} = A \cos t - B \sin t$

invullen geeft:  $\frac{dy}{dt} = -3(A \sin t + B \cos t) - \sin t$   
 $= -3A \sin t - 3B \cos t - \sin t$   
 $= (-3A - 1) \sin t - 3B \cos t$

$$\begin{cases} -3A - 1 = -B \\ -3B = A \end{cases}$$

$$-3 \cdot -3B - 1 = -B$$

$$9B - 1 = -B$$

$$10B = 1$$

$$B = \frac{1}{10}$$

$$A = -\frac{3}{10}$$

$y = -\frac{3}{10} \sin t + \frac{1}{10} \cos t$  is een particuliere oplossing

dus  $y = c \cdot e^{-3t} - \frac{3}{10} \sin t + \frac{1}{10} \cos t$

c.  $\frac{dy}{dt} = 4y - \cos t$

$\frac{dy}{dt} = 4y$  geeft  $y = c \cdot e^{4t}$

stel  $y = A \sin t + B \cos t$

differentiëren geeft:  $\frac{dy}{dt} = A \cos t - B \sin t$

invullen geeft:  $\frac{dy}{dt} = 4(A \sin t + B \cos t) - \cos t$

$$= 4A \sin t + 4B \cos t - \cos t$$

$$= 4A \sin t + (4B - 1) \cos t$$

$$\begin{cases} 4A = -B \\ 4B - 1 = A \end{cases}$$

$$4(4B - 1) = -B$$

$$16B - 4 = -B$$

$$17B = 4$$

$$B = \frac{4}{17}$$

$$A = -\frac{1}{17}$$

$y = -\frac{1}{17} \sin t + \frac{4}{17} \cos t$  is een particuliere oplossing

dus  $y = c \cdot e^{4t} - \frac{1}{17} \sin t + \frac{4}{17} \cos t$

d.  $\frac{dy}{dt} = \frac{1}{2}y + 2 \sin t - 3 \cos t$

$$\frac{dy}{dt} = \frac{1}{2}y \text{ geeft } y = c \cdot e^{\frac{1}{2}t}$$

stel  $y = A \sin t + B \cos t$

differentiëren geeft:  $\frac{dy}{dt} = A \cos t - B \sin t$

invullen geeft:  $\frac{dy}{dt} = \frac{1}{2}(A \sin t + B \cos t) + 2 \sin t - 3 \cos t$

$$= \frac{1}{2}A \sin t + \frac{1}{2}B \cos t + 2 \sin t - 3 \cos t$$

$$= (\frac{1}{2}A + 2) \sin t + (\frac{1}{2}B - 3) \cos t$$

$$\begin{cases} \frac{1}{2}A + 2 = -B \\ \frac{1}{2}B - 3 = A \end{cases}$$

$$\frac{1}{2}(\frac{1}{2}B - 3) + 2 = -B$$

$$\frac{1}{4}B - 1\frac{1}{2} + 2 = -B$$

$$1\frac{1}{4}B = -\frac{1}{2}$$

$$B = -\frac{2}{5}$$

$$A = -3\frac{1}{5}$$

$y = -3\frac{1}{5} \sin t - \frac{2}{5} \cos t$  is een particuliere oplossing

dus  $y = c \cdot e^{\frac{1}{2}t} - 3\frac{1}{5} \sin t - \frac{2}{5} \cos t$

### **Opgave 47:**

a.  $\frac{dy}{dt} = 2y - 3e^t$

$$\frac{dy}{dt} = 2y \text{ geeft } y = c \cdot e^{2t}$$

stel  $y = A \cdot e^t$

differentiëren geeft:  $\frac{dy}{dt} = A \cdot e^t$

invullen geeft:  $\frac{dy}{dt} = 2A \cdot e^t - 3e^t = (2A - 3)e^t$

$$2A - 3 = A$$

$$A = 3$$

$y = 3e^t$  is een particuliere oplossing

dus  $y = c \cdot e^{2t} + 3e^t$

b.  $\frac{dy}{dt} = 10y - 5e^t$

$$\frac{dy}{dt} = 10y \text{ geeft } y = c \cdot e^{10t}$$

$$\text{stel } y = A \cdot e^t$$

$$\text{differentiëren geeft: } \frac{dy}{dt} = A \cdot e^t$$

$$\text{invullen geeft: } \frac{dy}{dt} = 10A \cdot e^t - 5e^t = (10A - 5)e^t$$

$$10A - 5 = A$$

$$9A = 5$$

$$A = \frac{5}{9}$$

$y = \frac{5}{9}e^t$  is een particuliere oplossing

$$\text{dus } y = c \cdot e^{10t} + \frac{5}{9}e^t$$

c.  $\frac{dy}{dt} = -2y + e^t$

$$\frac{dy}{dt} = -2y \text{ geeft } y = c \cdot e^{-2t}$$

$$\text{stel } y = A \cdot e^t$$

$$\text{differentiëren geeft: } \frac{dy}{dt} = A \cdot e^t$$

$$\text{invullen geeft: } \frac{dy}{dt} = -2A \cdot e^t + e^t = (-2A + 1)e^t$$

$$-2A + 1 = A$$

$$-3A = -1$$

$$A = \frac{1}{3}$$

$y = \frac{1}{3}e^t$  is een particuliere oplossing

$$\text{dus } y = c \cdot e^{-2t} + \frac{1}{3}e^t$$

d.  $\frac{dy}{dt} = 1\frac{1}{2}y + 2e^t$

$$\frac{dy}{dt} = 1\frac{1}{2}y \text{ geeft } y = c \cdot e^{1\frac{1}{2}t}$$

$$\text{stel } y = A \cdot e^t$$

$$\text{differentiëren geeft: } \frac{dy}{dt} = A \cdot e^t$$

$$\text{invullen geeft: } \frac{dy}{dt} = 1\frac{1}{2}A \cdot e^t + 2e^t = (1\frac{1}{2}A + 2)e^t$$

$$1\frac{1}{2}A + 2 = A$$

$$\frac{1}{2}A = -2$$

$$A = -4$$

$y = -4e^t$  is een particuliere oplossing

$$\text{dus } y = c \cdot e^{1\frac{1}{2}t} - 4e^t$$

### **Opgave 48:**

a.  $\frac{dy}{dt} = 2y + 4t - 2$

$$\frac{dy}{dt} = 2y \text{ geeft } y = c \cdot e^{2t}$$

$$\text{stel } y = at + b$$

$$\text{differentiëren geeft: } \frac{dy}{dt} = a$$

$$\begin{aligned} \text{invullen geeft: } \frac{dy}{dt} &= 2(at + b) + 4t - 2 \\ &= 2at + 2b + 4t - 2 \\ &= (2a + 4)t + 2b - 2 \end{aligned}$$

$$\begin{cases} 2a + 4 = 0 \\ 2b - 2 = a \end{cases}$$

$$2a = -4$$

$$a = -2$$

$$2b - 2 = -2$$

$$b = 0$$

$y = -2t$  is een particuliere oplossing

$$\text{dus } y = c \cdot e^{2t} - 2t$$

b.  $\frac{dy}{dt} = \frac{1}{2}y + 4e^{2t}$

$$\frac{dy}{dt} = \frac{1}{2}y \text{ geeft } y = c \cdot e^{\frac{1}{2}t}$$

$$\text{stel } y = A \cdot e^{2t}$$

$$\text{differentiëren geeft: } \frac{dy}{dt} = 2A \cdot e^{2t}$$

$$\text{invullen geeft: } \frac{dy}{dt} = \frac{1}{2}A \cdot e^{2t} + 4e^{2t} = \left(\frac{1}{2}A + 4\right) \cdot e^{2t}$$

$$\frac{1}{2}A + 4 = 2A$$

$$-1\frac{1}{2}A = -4$$

$$A = 2\frac{2}{3}$$

$y = 2\frac{2}{3}e^{2t}$  is een particuliere oplossing

$$\text{dus } y = c \cdot e^{\frac{1}{2}t} + 2\frac{2}{3}e^{2t}$$

c.  $\frac{dy}{dt} = -3y + \sin(2t)$

$$\frac{dy}{dt} = -3y \text{ geeft } y = c \cdot e^{-3t}$$

$$\text{stel } y = A \sin(2t) + B \cos(2t)$$

$$\text{differentiëren geeft: } \frac{dy}{dt} = 2A \cos(2t) - 2B \sin(2t)$$

$$\text{invullen geeft: } \frac{dy}{dt} = -3(A \sin(2t) + B \cos(2t)) + \sin(2t)$$

$$= -3A \sin(2t) - 3B \cos(2t) + \sin(2t)$$

$$= (-3A + 1) \sin(2t) - 3B \cos(2t)$$

$$\begin{cases} -3A + 1 = -2B \\ -3B = 2A \end{cases}$$

$$A = -1\frac{1}{2}B$$

$$4\frac{1}{2}B + 1 = -2B$$

$$6\frac{1}{2}B = -1$$

$$B = -\frac{2}{13}$$

$$A = \frac{3}{13}$$

$y = \frac{3}{13} \sin(2t) - \frac{2}{13} \cos(2t)$  is een particuliere oplossing

$$\text{dus } y = c \cdot e^{-3t} + \frac{3}{13} \sin(2t) - \frac{2}{13} \cos(2t)$$

d.  $\frac{dy}{dt} = -2y + 2t^2 - 4t$

$$\frac{dy}{dt} = -2y \text{ geeft } y = c \cdot e^{-2t}$$

$$\text{stel } y = at^2 + bt + c$$

$$\text{differentiëren geeft: } \frac{dy}{dt} = 2at + b$$

$$\text{invullen geeft: } \frac{dy}{dt} = -2(at^2 + bt + c) + 2t^2 - 4t$$

$$= -2at^2 - 2bt - 2c + 2t^2 - 4t$$

$$= (-2a + 2)t^2 + (-2b - 4)t - 2c$$

$$\begin{cases} -2a + 2 = 0 \\ -2b - 4 = 2a \\ -2c = b \end{cases}$$

$$-2a = -2$$

$$a = 1$$

$$-2b - 4 = 2$$

$$-2b = 6$$

$$b = -3$$

$$-2c = -3$$

$$c = 1\frac{1}{2}$$

$y = t^2 - 3t + 1\frac{1}{2}$  is een particuliere oplossing

$$\text{dus } y = c \cdot e^{-2t} + t^2 - 3t + 1\frac{1}{2}$$

### **Opgave 49:**

a.  $\frac{dy}{dt} = -y - 3$

$y = -3$  is een particuliere oplossing

$$\frac{dy}{dt} = -y \text{ geeft } \frac{dy}{dt} = c \cdot e^{-t}$$

$$\text{dus } y = c \cdot e^{-t} - 3$$

$$y(0) = c - 3 = 1$$

$$c = 4$$

$$y(t) = 4e^{-t} - 3$$

b.  $y(0) = c - 3 = 0$

$$c = 3$$

$$y(t) = 3e^{-t} - 3$$

$$y(1) = 3e^{-1} - 3 = -3 + \frac{3}{e} \text{ dus het punt } (1, -3 + \frac{3}{e}) \text{ ligt op de oplossingskromme}$$

c.  $y(3) = c \cdot e^{-3} - 3 = 1$

$$c \cdot e^{-3} = 4$$

$$c = 4e^3$$

$$y(t) = 4e^3 \cdot e^{-t} - 3 = 4e^{3-t} - 3$$

$$y(a) = 4e^{3-a} - 3 = 8$$

$$4e^{3-a} = 11$$

$$e^{3-a} = 2\frac{3}{4}$$

$$3 - a = \ln(2\frac{3}{4})$$

$$a = 3 - \ln(2\frac{3}{4})$$

### **Opgave 50:**

a.  $\frac{dT}{dt} = -0,035(T - 18)$

$T = 18$  is een particuliere oplossing

$$\frac{dT}{dt} = -0,035T \text{ geeft } T = c \cdot e^{-0,035t}$$

$$T = c \cdot e^{-0,035t} + 18$$

$$T(0) = c + 18 = 5$$

$$c = -13$$

$$T = -13e^{-0,035t} + 18$$

b.  $T(5) = -13e^{-0,175} + 18 = 7^\circ\text{C}$

c.  $-13e^{-0,035t} + 18 = 15$

$$-13e^{-0,035t} = -3$$

$$e^{-0,035t} = \frac{3}{13}$$

$$-0,035t = \ln\left(\frac{3}{13}\right)$$

$$t = \frac{\ln\left(\frac{3}{13}\right)}{-0,035} = 42$$

### **Opgave 51:**

a.  $\frac{dT}{dt} = -0,03(T - 150)$

$T = 150$  is een particuliere oplossing

$$\frac{dT}{dt} = -0,03T \text{ geeft } T = c \cdot e^{-0,03t}$$

$$T = c \cdot e^{-0,03t} + 150$$

$$T(0) = c + 150 = 20$$

$$c = -130$$

$$T = -130 \cdot e^{-0,03t} + 150$$

$$-130e^{-0,03t} + 150 = 100$$

$$-130e^{-0,03t} = -50$$

$$e^{-0,03t} = \frac{5}{13}$$

$$-0,03t = \ln\left(\frac{5}{13}\right)$$

$$t = \frac{\ln\left(\frac{5}{13}\right)}{-0,03} = 31,9 \text{ dus na 32 minuten}$$

b.  $\frac{dT}{dt} = -0,001(T - 150)$

$T = 150$  is een particuliere oplossing

$$\frac{dT}{dt} = -0,001T \text{ geeft } T = c \cdot e^{-0,001t}$$

$$T = c \cdot e^{-0,001t} + 150$$

$$T(32) = c \cdot e^{-0,032} + 150 = 100$$

$$c \cdot e^{-0,032} = -50$$

$$c = -50e^{0,032}$$

$$T = -50e^{0,032} \cdot e^{-0,001t} + 150 = -50e^{0,032-0,001t} + 150$$

$$-50e^{0,032-0,001t} + 150 = 120$$

$$-50e^{0,032-0,001t} = -30$$

$$e^{0,032-0,001t} = 0,6$$

$$0,032 - 0,001t = \ln(0,6)$$

$$-0,001t = \ln(0,6) - 0,032$$

$$t = 543$$

dus het duurt nog ongeveer  $543 - 32 = 511$  minuten

### **Opgave 52:**

a.  $\frac{dN}{dt} = 0,7N$  geeft  $N = c \cdot e^{0,7t}$

$$N(0) = c = 1000$$

$$N = 1000e^{0,7t}$$

b.  $\left[\frac{dN}{dt}\right]_{N=10000} = 6825$   
 $\left[\frac{dN}{dt}\right]_{N=200000} = 70000$   
 $\left[\frac{dN}{dt}\right]_{N=390000} = 6825$

c.  $\frac{dN}{dt} = 0,7N \cdot \frac{400000 - N}{400000} = \frac{0,7N}{400000} \cdot (400000 - N) = 1,75 \cdot 10^{-6} \cdot (400000 - N)$

### **Opgave 53:**

a.  $y = \frac{1}{u}$  dan  $\frac{dy}{du} = -\frac{1}{u^2}$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt}$$

b.  $\frac{dy}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt}$  dus  $\frac{du}{dt} = -u^2 \cdot \frac{dy}{dt}$

$$y = \frac{1}{u} \text{ dus } u = \frac{1}{y}$$

invullen geeft:  $\frac{du}{dt} = -\frac{1}{y^2} \cdot y(1-y) = -\frac{1}{y} + 1 = -\left(\frac{1}{y} - 1\right) = -(u-1)$

$u = 1$  is een particuliere oplossing

$$\frac{du}{dt} = -u \text{ geeft } u = c \cdot e^{-t}$$

$$u = c \cdot e^{-t} + 1$$

c.  $y = \frac{1}{u} = \frac{1}{1 + c \cdot e^{-t}}$

### **Opgave 54:**

a.  $\frac{dy}{dt} = y^2 + 4y$

stel  $y = \frac{1}{u}$  dan  $\frac{dy}{du} = -\frac{1}{u^2}$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = \frac{1}{u^2} + \frac{4}{u}$$

$$\frac{du}{dt} = -1 - 4u$$

$u = -\frac{1}{4}$  is een particuliere oplossing

$$\frac{du}{dt} = -4u \text{ geeft } u = c \cdot e^{-4t}$$

$$u = -\frac{1}{4} + c \cdot e^{-4t}$$

$$y = \frac{1}{-\frac{1}{4} + c \cdot e^{-4t}} = \frac{-4}{1 + c_1 \cdot e^{-4t}}$$

b.  $\frac{dy}{dt} = \frac{1}{2}y(1-y) = \frac{1}{2}y - \frac{1}{2}y^2$

stel  $y = \frac{1}{u}$  dan  $\frac{dy}{du} = -\frac{1}{u^2}$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = \frac{1}{2u} - \frac{1}{2u^2}$$

$$\frac{du}{dt} = -\frac{1}{2}u + \frac{1}{2}$$

$u = 1$  is een particuliere oplossing

$$\frac{du}{dt} = -\frac{1}{2}u \text{ geeft } u = c \cdot e^{-\frac{1}{2}t}$$

$$u = 1 + c \cdot e^{-\frac{1}{2}t}$$

$$y = \frac{1}{1 + c \cdot e^{-\frac{1}{2}t}}$$

c.  $\frac{dy}{dt} = 0,1y(1 - \frac{y}{100}) = 0,1y - 0,001y^2$

stel  $y = \frac{1}{u}$  dan  $\frac{dy}{du} = -\frac{1}{u^2}$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = \frac{0,1}{u} - \frac{0,001}{u^2}$$

$$\frac{du}{dt} = -0,1u + 0,001$$

$u = 0,01$  is een particuliere oplossing

$$\frac{du}{dt} = -0,1u \text{ geeft } u = c \cdot e^{-0,1t}$$

$$u = 0,01 + c \cdot e^{-0,1t}$$

$$y = \frac{1}{0,01 + c \cdot e^{-0,1t}} = \frac{100}{1 + c_1 \cdot e^{-0,1t}}$$

d.  $\frac{dA}{dt} = 0,001A(1 - \frac{A}{20}) = 0,001A - 0,00005A^2$

stel  $A = \frac{1}{u}$  dan  $\frac{dA}{du} = -\frac{1}{u^2}$

$$\frac{dA}{dt} = \frac{dA}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 0,001 \cdot \frac{1}{u} - 0,00005 \cdot \frac{1}{u^2}$$

$$\frac{du}{dt} = -0,001u + 0,00005$$

$u = 0,05$  is een particuliere oplossing

$$\frac{du}{dt} = -0,001u \text{ geeft } u = c \cdot e^{-0,001t}$$

$$u = 0,05 + c \cdot e^{-0,001t}$$

$$A = \frac{1}{0,05 + c \cdot e^{-0,001t}} = \frac{20}{1 + c_1 \cdot e^{-0,001t}}$$

### **Opgave 55:**

a.  $\frac{dy}{dt} = 2y^2 - 10y$

stel  $y = \frac{1}{u}$  dan  $\frac{dy}{du} = -\frac{1}{u^2}$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 2 \cdot \frac{1}{u^2} - 10 \cdot \frac{1}{u}$$

$$\frac{du}{dt} = -2 + 10u$$

$u = \frac{1}{5}$  is een particuliere oplossing

$$\frac{du}{dt} = 10u \text{ geeft } u = c \cdot e^{10t}$$

$$u = \frac{1}{5} + c \cdot e^{10t}$$

$$y = \frac{1}{\frac{1}{5} + c \cdot e^{10t}} = \frac{5}{1 + c_1 \cdot e^{10t}}$$

b.  $\frac{dy}{dt} = \frac{1}{2}y + 4e^t - 2\sin t$

$$\frac{dy}{dt} = \frac{1}{2}y \text{ geeft } y = c \cdot e^{\frac{1}{2}t}$$

stel  $y = C \cdot e^t + A \sin t + B \cos t$

differentiëren geeft:  $\frac{dy}{dt} = C \cdot e^t + A \cos t - B \sin t$

invullen geeft:  $\frac{dy}{dt} = \frac{1}{2}(C \cdot e^t + A \sin t + B \cos t) + 4e^t - 2\sin t$   
 $= \frac{1}{2}C \cdot e^t + \frac{1}{2}A \sin t + \frac{1}{2}B \cos t + 4e^t - 2\sin t$   
 $= (\frac{1}{2}C + 4) \cdot e^t + (\frac{1}{2}A - 2)\sin t + \frac{1}{2}B \cos t$

$$\begin{cases} \frac{1}{2}C + 4 = C \\ \frac{1}{2}A - 2 = -B \\ \frac{1}{2}B = A \end{cases}$$

$$-\frac{1}{2}C = -4$$

$$C = 8$$

$$\frac{1}{2} \cdot \frac{1}{2}B - 2 = -B$$

$$1\frac{1}{4}B = 2$$

$$B = 1\frac{3}{5}$$

$$A = \frac{4}{5}$$

$y = 8e^t + \frac{4}{5}\sin t + 1\frac{3}{5}\cos t$  is een particuliere oplossing

dus  $y = c \cdot e^{\frac{1}{2}t} + 8e^t + \frac{4}{5}\sin t + 1\frac{3}{5}\cos t$

c.  $\frac{dy}{dt} = 3ty + 6t$

$$\frac{dy}{dt} = 3t(y + 2)$$

$$\frac{1}{y+2} dy = 3t dt$$

$$\ln(y+2) = 1\frac{1}{2}t^2 + c$$

$$y+2 = e^{\frac{1}{2}t^2+c} = c_1 \cdot e^{\frac{1}{2}t^2}$$

$$y = -2 + c_1 \cdot e^{\frac{1}{2}t^2}$$

d.  $\frac{dy}{dt} = -2y + 12y^2$

stel  $y = \frac{1}{u}$  dan  $\frac{dy}{du} = -\frac{1}{u^2}$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = -2 \cdot \frac{1}{u} + 12 \cdot \frac{1}{u^2}$$

$$\frac{du}{dt} = 2u - 12$$

$u = 6$  is een particuliere oplossing

$$\frac{du}{dt} = 2u \text{ geeft } u = c \cdot e^{2t}$$

$$u = 6 + c \cdot e^{2t}$$

$$y = \frac{1}{6 + c \cdot e^{2t}}$$

**Opgave 56:**

a.  $\frac{dy}{dt} = y - 2y^2$

stel  $y = \frac{1}{u}$  dan  $\frac{dy}{du} = -\frac{1}{u^2}$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = \frac{1}{u} - \frac{2}{u^2}$$

$$\frac{du}{dt} = -u + 2$$

 $u = 2$  is een particuliere oplossing

$\frac{du}{dt} = -u$  geeft  $u = c \cdot e^{-t}$

$u = 2 + c \cdot e^{-t}$

$$y = \frac{1}{2 + c \cdot e^{-t}}$$

$$y(0) = \frac{1}{2 + c} = \frac{1}{5}$$

$c = 3$

$$y = \frac{1}{2 + 3e^{-t}}$$

b.  $y(0) = \frac{1}{2 + c} = 2$

$2(2 + c) = 1$

$4 + 2c = 1$

$2c = -3$

$c = -1\frac{1}{2}$

$$y = \frac{1}{2 - 1\frac{1}{2}e^{-t}}$$

$$y(1) = \frac{1}{2 - 1\frac{1}{2}e^{-1}} = \frac{e}{2e - 1\frac{1}{2}} = \frac{2e}{4e - 3}$$

**Opgave 57:**

a.  $\frac{dN}{dt} = c \cdot N \cdot (10000 - N)$

$\left[\frac{dN}{dt}\right]_{N=800} = 800c(10000 - 800) = 350$

$c = 4,755 \cdot 10^{-5}$

$\frac{dN}{dt} = 4,755 \cdot 10^{-5} \cdot N \cdot (10000 - N)$

$$= 0,4755N - 4,755 \cdot 10^{-5} \cdot N^2$$

stel  $N = \frac{1}{u}$  dan  $\frac{dN}{du} = -\frac{1}{u^2}$

$$\frac{dN}{dt} = \frac{dN}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 0,4755 \cdot \frac{1}{u} - 4,755 \cdot 10^{-5} \cdot \frac{1}{u^2}$$

$$\frac{du}{dt} = -0,4755u + 4,755 \cdot 10^{-5}$$

 $u = 0,0001$  is een particuliere oplossing

$\frac{du}{dt} = -0,4755u$  geeft  $u = c \cdot e^{-0,4755t}$

$u = 0,0001 + c \cdot e^{-0,4755t}$

$$y = \frac{1}{0,0001 + c \cdot e^{-0,4755t}} = \frac{10000}{1 + c_1 \cdot e^{-0,4755t}}$$

$$y(0) = \frac{10000}{1 + c} = 800$$

$$800 + 800c = 10000$$

$$800c = 9200$$

$$c = 11,5$$

$$y = \frac{10000}{1 + 11,5e^{-0,4755t}}$$

$$y(20) = 9991$$

b.  $y = \frac{10000}{1 + 11,5e^{-0,4755t}} = 5000$

neem  $y_1 = \frac{10000}{1 + 11,5e^{-0,4755x}}$  en  $y_2 = 5000$

intersect geeft  $x = 5,14$

dus vanaf  $t = 5,14$  zijn er meer dan 5000 zeeotters

### Opgave 58:

a.  $\frac{dL}{dt} = 6L - 0,2L^2$

stel  $L = \frac{1}{u}$  dan  $\frac{dL}{du} = -\frac{1}{u^2}$

$$\frac{dL}{dt} = \frac{dL}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = \frac{6}{u} - 0,2 \cdot \frac{1}{u^2}$$

$$\frac{du}{dt} = -6u + 0,2$$

$u = \frac{1}{30}$  is een particuliere oplossing

$$\frac{du}{dt} = -6u \text{ geeft } u = c \cdot e^{-6t}$$

$$u = \frac{1}{30} + c \cdot e^{-6t}$$

$$L = \frac{1}{\frac{1}{30} + c \cdot e^{-t}} = \frac{30}{1 + c_1 \cdot e^{-6t}}$$

$$L(0) = \frac{30}{1 + c_1} = 2$$

$$c_1 = 14$$

$$L = \frac{30}{1 + 14e^{-6t}}$$

$$L\left(\frac{1}{7}\right) - L(0) = 4,32 - 2 = 2,32 \text{ cm}$$

b. de grenswaarde is 30 cm

$$L = \frac{30}{1 + 14e^{-6t}} = 29$$

neem  $y_1 = \frac{30}{1 + 14e^{-6x}}$  en  $y_2 = 29$

intersect geeft  $x = 1,01$  dus na 7 dagen

**Opgave 59:**

a.  $\frac{dN}{dt} = 0,00006N(5000 - N)$   
 $= 0,3N - 0,00006N^2$

stel  $N = \frac{1}{u}$  dan  $\frac{dN}{du} = -\frac{1}{u^2}$

$$\frac{dN}{dt} = \frac{dN}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 0,3 \cdot \frac{1}{u} - 0,00006 \cdot \frac{1}{u^2}$$

$$\frac{du}{dt} = -0,3u + 0,00006$$

$u = 0,0002$  is een particuliere oplossing

$$\frac{du}{dt} = -0,3u \text{ geeft } u = c \cdot e^{-0,3t}$$

$$u = 0,0002 + c \cdot e^{-0,3t}$$

$$N = \frac{1}{0,0002 + c \cdot e^{-0,3t}} = \frac{5000}{1 + c_1 \cdot e^{-0,3t}}$$

$$N(0) = \frac{5000}{1 + c_1} = 4500$$

$$4500 + 4500c_1 = 5000$$

$$4500c_1 = 500$$

$$c_1 = \frac{1}{9}$$

$$N = \frac{5000}{1 + \frac{1}{9}e^{-0,3t}}$$

$$N(10) = 4972$$

b. de grenswaarde wordt nu dus 2500

$$\frac{dN}{dt} = 0,00004N(2500 - N)$$
  
 $= 0,1N - 0,00004N^2$

stel  $N = \frac{1}{u}$  dan  $\frac{dN}{du} = -\frac{1}{u^2}$

$$\frac{dN}{dt} = \frac{dN}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 0,1 \cdot \frac{1}{u} - 0,00004 \cdot \frac{1}{u^2}$$

$$\frac{du}{dt} = -0,1u + 0,00004$$

$u = 0,0004$  is een particuliere oplossing

$$\frac{du}{dt} = -0,1u \text{ geeft } u = c \cdot e^{-0,1t}$$

$$u = 0,0004 + c \cdot e^{-0,1t}$$

$$N = \frac{1}{0,0004 + c \cdot e^{-0,1t}} = \frac{2500}{1 + c_1 \cdot e^{-0,1t}}$$

$$N(10) = \frac{2500}{1 + c_1 \cdot e^{-1}} = 4972$$

$$4972 + 4972c_1 \cdot e^{-1} = 2500$$

$$4972c_1 \cdot e^{-1} = -2472$$

$$c_1 = 1,351$$

$$N = \frac{2500}{1 - 1,351e^{-0,1t}}$$

$$\frac{2500}{1-1,351e^{-0,1t}} = 4500$$

neem  $y_1 = \frac{2500}{1-1,351e^{-0,1x}}$  en  $y_2 = 4500$

intersect geeft  $x = 11,1$

dus  $11,1 - 10 = 1,1$  jaar na  $t = 10$  zijn er weer 4500 panda's