15.5 Differentiaalvergelijkingen van de tweede orde

Opgave 60:

a. de tweede afgeleide is de hoogste afgeleide die in de differentiaalvergelijking voorkomt

b. $y(t) = e^{\lambda t}$

$y' = \lambda \cdot e^{\lambda t}$

$y'' = \lambda^2 \cdot e^{\lambda t}$

invullen in de dv geeft:

$\lambda^2 \cdot e^{\lambda t} + 5\lambda \cdot e^{\lambda t} + 4e^{\lambda t} = 0$

$(\lambda^2 + 5\lambda + 4) \cdot e^{\lambda t} = 0$

$\lambda^2 + 5\lambda + 4 = 0 \quad \vee \quad e^{\lambda t} = 0$ (k.n.)

dus $\lambda^2 + 5\lambda + 4 = 0$

$(\lambda + 4)(\lambda + 1) = 0$

$\lambda = -4 \quad \vee \quad \lambda = -1$

c. $y(t) = A \cdot e^{-t} + B \cdot e^{-4t}$

$y' = -A \cdot e^{-t} - 4B \cdot e^{-4t}$

$y'' = A \cdot e^{-t} + 16B \cdot e^{-4t}$

invullen in de dv geeft:

$A \cdot e^{-t} + 16B \cdot e^{-4t} - 5A \cdot e^{-t} - 20B \cdot e^{-4t} + 4A \cdot e^{-t} + B \cdot e^{-4t} = 0$

$0 = 0$ dus klopt

dus voor elke waarde van A en B voldoet $y(t) = A \cdot e^{-t} + B \cdot e^{-4t}$ aan de dv

Opgave 61:

a. $y'' + 4y' + 5y = 0$

$y = e^{\lambda t}$ dus $y' = \lambda \cdot e^{\lambda t}$ en $y'' = \lambda^2 \cdot e^{\lambda t}$

invullen in de dv geeft:

$\lambda^2 \cdot e^{\lambda t} + 4\lambda \cdot e^{\lambda t} + 5e^{\lambda t} = 0$

$(\lambda^2 + 4\lambda + 5) \cdot e^{\lambda t} = 0$

$\lambda^2 + 4\lambda + 5 = 0 \quad \vee \quad e^{\lambda t} = 0$ (k.n.)

$D = 4^2 - 4 \cdot 1 \cdot 5 = -4$ dus geen oplossingen

b. $\lambda^2 + 4\lambda + 5 = 0$

$(\lambda + 2)^2 - 4 + 5 = 0$

$(\lambda + 2)^2 = -1$

$\lambda + 2 = i \quad \vee \quad \lambda + 2 = -i$

$\lambda = -2 + i \quad \vee \quad \lambda = -2 - i$

Opgave 62:

λ_1 is oplossing van $\lambda^2 + p\lambda + q = 0$ dus geldt: ${\lambda_1}^2 + p\lambda_1 + q = 0$

λ_2 is ook een oplossing dus geldt: ${\lambda_2}^2 + p\lambda_2 + q = 0$

als $y = A \cdot e^{\lambda_1 t} + B \cdot e^{\lambda_2 t}$

dan $y' = \lambda_1 A \cdot e^{\lambda_1 t} + \lambda_2 B \cdot e^{\lambda_2 t}$

en $y'' = {\lambda_1}^2 A \cdot e^{\lambda_1 t} + {\lambda_2}^2 B \cdot e^{\lambda_2 t}$

invullen in de dv geeft:

$\lambda_1{}^2 A\cdot e^{\lambda_1 t} + \lambda_2{}^2 B\cdot e^{\lambda_2 t} + p(\lambda_1 A\cdot e^{\lambda_1 t} + \lambda_2 B\cdot e^{\lambda_2 t}) + q(A\cdot e^{\lambda_1 t} + B\cdot e^{\lambda_2 t}) = 0$

$(\lambda_1{}^2 + p\lambda_1 + q)\cdot A\cdot e^{\lambda_1 t} + (\lambda_2{}^2 + p\lambda_2 + q)\cdot B\cdot e^{\lambda_2 t} = 0$

$0 = 0$ klopt, dus alle functies van de vorm $y(t) = A\cdot e^{\lambda_1 t} + B\cdot e^{\lambda_2 t}$ zijn oplossing van de dv

Opgave 63:

λ_1 is oplossing van $\lambda^2 + p\lambda + q = 0$ dus geldt: $\lambda_1{}^2 + p\lambda_1 + q = 0$ en omdat het de enige reële oplossing is geldt dat: $x_{top} = \frac{-p}{2} = -\frac{1}{2}p = \lambda_1$ dus $\lambda_1 + \frac{1}{2}p = 0$ ofwel $2\lambda_1 + p = 0$

stel $y = (A + Bt)\cdot e^{\lambda_1 t}$

dan $y' = B\cdot e^{\lambda_1 t} + (A + Bt)\cdot \lambda_1 \cdot e^{\lambda_1 t}$

en $y'' = B\cdot\lambda_1\cdot e^{\lambda_1 t} + B\cdot\lambda_1\cdot e^{\lambda_1 t} + (A + Bt)\cdot\lambda_1{}^2\cdot e^{\lambda_1 t}$

$\quad = 2B\cdot\lambda_1\cdot e^{\lambda_1 t} + (A + Bt)\cdot\lambda_1{}^2\cdot e^{\lambda_1 t}$

invullen in de dv geeft:

$2B\cdot\lambda_1\cdot e^{\lambda_1 t} + (A + Bt)\cdot\lambda_1{}^2\cdot e^{\lambda_1 t} + p(B\cdot e^{\lambda_1 t} + (A + Bt)\cdot\lambda_1\cdot e^{\lambda_1 t}) + q(A + Bt)\cdot e^{\lambda_1 t} = 0$

$(\lambda_1{}^2 + p\lambda_1 + q)(A + Bt)\cdot e^{\lambda_1 t} + B(2\lambda_1 + p)\cdot e^{\lambda_1 t} = 0$

$0\cdot(A + Bt)\cdot e^{\lambda_1 t} + B\cdot 0\cdot e^{\lambda_1 t} = 0$

$0 = 0$ klopt, dus alle functies van de vorm $y = (A + Bt)\cdot e^{\lambda_1 t}$ zijn oplossing van de dv

Opgave 64:

$\lambda_1 = a + bi$ is oplossing van $\lambda^2 + p\lambda + q = 0$ dus $(a + bi)^2 + p(a + bi) + q = 0$

$a^2 + 2abi - b^2 + pa + pbi + q = 0$

$a^2 - b^2 + pa + q + b(2a + p)i = 0$

$a^2 - b^2 + pa + q = 0 \quad \wedge \quad 2a + p = 0$ want $b \neq 0$

$a^2 - b^2 + pa + q = 0 \quad \wedge \quad p = -2a$

$a^2 - b^2 - 2a^2 + q = 0 \quad \wedge \quad p = -2a$

$q = a^2 + b^2 \quad \wedge \quad p = -2a$

$\lambda_2 = a - bi$ is oplossing van $\lambda^2 + p\lambda + q = 0$ dus $(a - bi)^2 + p(a - bi) + q = 0$

$a^2 - 2abi - b^2 + pa - pbi + q = 0$

$a^2 - b^2 + pa + q - b(2a + p)i = 0$

$a^2 - b^2 + pa + q = 0 \quad \wedge \quad 2a + p = 0$ want $b \neq 0$

$a^2 - b^2 + pa + q = 0 \quad \wedge \quad p = -2a$

$a^2 - b^2 - 2a^2 + q = 0 \quad \wedge \quad p = -2a$

$q = a^2 + b^2 \quad \wedge \quad p = -2a$

stel $y(t) = (A\cos(bt) + B\sin(bt))\cdot e^{at}$

dus $y' = (-bA\sin(bt) + bB\cos(bt))\cdot e^{at} + (A\cos(bt) + B\sin(bt))\cdot ae^{at}$

$\quad = (-bA\sin(bt) + bB\cos(bt) + aA\cos(bt) + aB\sin(bt))\cdot e^{at}$

dus $y'' = (-b^2 A\cos(bt) - b^2 B\sin(bt) - abA\sin(bt) + abB\cos(bt)\cdot e^{at} +$

$\quad (-bA\sin(bt) + bB\cos(bt) + aA\cos(bt) + aB\sin(bt))\cdot ae^{at}$

$\quad = (a^2 A\cos(bt) - b^2 A\cos(bt) + a^2 B\sin(bt) - b^2 B\sin(bt) + 2abB\cos(bt) - 2abA\sin(bt))\cdot e^{at}$

invullen in de dv geeft:

$(a^2 A\cos(bt) - b^2 A\cos(bt) + a^2 B\sin(bt) - b^2 B\sin(bt) + 2abB\cos(bt) - 2abA\sin(bt)) \cdot e^{at}$ +
$-2a(-bA\sin(bt) + bB\cos(bt) + aA\cos(bt) + aB\sin(bt)) \cdot e^{at}$ +
$(a^2 + b^2)(A\cos(bt) + B\sin(bt)) \cdot e^{at} = 0$
haakjes wegwerken levert nu $0 = 0$ dus klopt

Opgave 65:

a. $y'' + 6y' + 8 = 0$

$\lambda^2 + 6\lambda + 8 = 0$

$(\lambda + 4)(\lambda + 2) = 0$

$\lambda = -4 \quad \vee \quad \lambda = -2$

dus $y = A \cdot e^{-4t} + B \cdot e^{-2t}$ en dus $y' = -4A \cdot e^{-4t} - 2B \cdot e^{-2t}$

$y(0) = 1$ dus $A + B = 1$

$y'(0) = -1$ dus $-4A - 2B = -1$

$$\begin{cases} A + B = 1 & \times 2 \\ -4A - 2B = -1 & \times 1 \end{cases}$$

$$\begin{cases} 2A + 2B = 2 \\ -4A - 2B = -1 \quad + \end{cases}$$

$-2A = 1$

$A = -\frac{1}{2}$

$B = 1\frac{1}{2}$

$y(t) = -\frac{1}{2}e^{-4t} + 1\frac{1}{2}e^{-2t}$

b. $y'' + 6y' + 10y = 0$

$\lambda^2 + 6\lambda + 10 = 0$

$(\lambda + 3)^2 - 9 + 10 = 0$

$(\lambda + 3)^2 = -1$

$\lambda + 3 = i \quad \vee \quad \lambda + 3 = -i$

$\lambda = -3 + i \quad \vee \quad \lambda = -3 - i$

dus $y(t) = (A\cos(t) + B\sin(t)) \cdot e^{-3t}$

dus $y' = (-A\sin(t) + B\cos(t)) \cdot e^{-3t} + (A\cos(t)) + B\sin(t)) \cdot -3e^{-3t}$

$= ((B - 3A)\cos(t) + (-A - 3B)\sin(t)) \cdot e^{-3t}$

$y(0) = 1$ geeft: $A = 1$

$y'(0) = -1$ geeft: $B - 3A = -1$

$B = 3A - 1 = 3 - 1 = 2$

dus $y(t) = (\cos(t) + 2\sin(t)) \cdot e^{-3t}$

c. $y'' - 8y' + 16 = 0$

$\lambda^2 - 8\lambda + 16 = 0$

$(\lambda - 4)^2 = 0$

$\lambda = 4$

dus $y(t) = (A + Bt) \cdot e^{4t}$

dus $y'(t) = B \cdot e^{4t} + (A + Bt) \cdot 4e^{4t}$

$y(0) = 1$ dus $A = 1$

$y'(0)=-1$ dus $B+4A=-1$
$B=-4A-1=-4-1=-5$
dus $y(t)=(1-5t)\cdot e^{4t}$

d. $y''+\frac{1}{4}y=0$
$\lambda^2+\frac{1}{4}=0$
$\lambda^2=-\frac{1}{4}$
$\lambda=\frac{1}{2}i \quad \vee \quad \lambda=-\frac{1}{2}i$
$y(t)=(A\cos(\frac{1}{2}t)+B\sin(\frac{1}{2}t))\cdot e^{0t}=A\cos(\frac{1}{2}t)+B\sin(\frac{1}{2}t)$
dus $y'(t)=-\frac{1}{2}A\sin(\frac{1}{2}t)+\frac{1}{2}B\cos(\frac{1}{2}t)$
$y(0)=1$ geeft: $A=1$
$y'(0)=-1$ geeft: $\frac{1}{2}B=-1$
dus $B=-2$
dus $y(t)=\cos(\frac{1}{2}t)-2\sin(\frac{1}{2}t)$

Opgave 66:

a. $x=-\frac{dy}{dt}+y$
differentiëren geeft: $\frac{dx}{dt}=-\frac{d^2y}{dt^2}+\frac{dy}{dt}$
invullen in $\frac{dx}{dt}=2x-6y$ geeft:
$-\frac{d^2y}{dt^2}+\frac{dy}{dt}=2\left(-\frac{dy}{dt}+y\right)-6y$
$-\frac{d^2y}{dt^2}+\frac{dy}{dt}=-2\frac{dy}{dt}+2y-6y$
$\frac{d^2y}{dt^2}-3\frac{dy}{dt}-4y=0$

b. $y'(0)=\left[\frac{dy}{dt}\right]_{t=0}=-x(0)+y(0)=-1-1=-2$

c. $y''-3y'-4y=0$
$\lambda^2-3\lambda-4=0$
$(\lambda-4)(\lambda+1)=0$
$\lambda=4 \quad \vee \quad \lambda=-1$
$y=A\cdot e^{4t}+B\cdot e^{-t}$ dus $y'=4A\cdot e^{4t}-B\cdot e^{-t}$
$y(0)=A+B=-1 \qquad y'(0)=4A-B=-2$

$$\begin{cases} A+B=1 \\ 4A-B=-2 \quad + \end{cases}$$

$5A=-1$
$A=-\frac{1}{5}$ dus $B=1\frac{1}{5}$
$y(t)=-\frac{1}{5}e^{4t}+1\frac{1}{5}e^{-t}$
$y'=-\frac{4}{5}e^{4t}-1\frac{1}{5}e^{-t}$
$x=-\frac{dy}{dt}+y$
$=-\left(-\frac{4}{5}e^{4t}-1\frac{1}{5}e^{-t}\right)+-\frac{1}{5}e^{4t}+1\frac{1}{5}e^{-t}$
$=\frac{4}{5}e^{4t}+1\frac{1}{5}e^{-t}-\frac{1}{5}e^{4t}+1\frac{1}{5}e^{-t}$
$=\frac{3}{5}e^{4t}+2\frac{2}{5}e^{-t}$

Opgave 67:

a. als je het blokje loslaat dan gaat het naar boven, dus F is naar boven gericht en dus positief, omdat $u < 0$ is $c > 0$

b. $F = m \cdot a = m \cdot \frac{dv}{dt}$

$\frac{dv}{dt} = \frac{F}{m} = \frac{-c \cdot u}{m}$

c. $\frac{dv}{dt} = \frac{-1 \cdot u}{1} = -u$

v is de snelheid waarmee u verandert, dus $\frac{du}{dt} = v$

het blokje wordt naar beneden getrokken tot $u = -1$ en wordt op $t = 0$ losgelaten

dus $u(0) = -1$ en $v(0) = 0$

d. $\frac{du}{dt} = v$ dus $\frac{d^2u}{dt^2} = \frac{dv}{dt}$

$\frac{dv}{dt} = -u$ dus $\frac{d^2u}{dt^2} = -u$

ofwel $\frac{d^2u}{dt^2} + u = 0$

$\lambda^2 + 1 = 0$

$\lambda^2 = -1$

$\lambda = i \quad \vee \quad \lambda = -i$

dus $u(t) = (A\cos t + B\sin t) \cdot e^{0t} = A\cos t + B\sin t$

$u(0) = A = -1$

$u'(t) = -A\sin t + B\cos t$

$u'(0) = B = 0$

dus $u(t) = -\cos t$

Opgave 68:

a. $F = -40\frac{du}{dt} - 72u$ en $\frac{d^2u}{dt^2} = \frac{F}{m} = \frac{F}{2} = \frac{1}{2}F$

$\frac{d^2u}{dt^2} = -20\frac{du}{dt} - 36u$

$\frac{d^2u}{dt^2} + 20\frac{du}{dt} + 36u = 0$

$\lambda^2 + 20\lambda + 36 = 0$

$(\lambda + 2)(\lambda + 18) = 0$

$\lambda = -2 \quad \vee \quad \lambda = -18$

$u(t) = A \cdot e^{-2t} + B \cdot e^{-18t}$

$u(0) = A + B = 1$

$u'(t) = -2A \cdot e^{-2t} - 18B \cdot e^{-18t}$

$v(0) = u'(0) = -2A - 18B = 2$

$$\begin{cases} A + B = 1 & \times 2 \\ -2A - 18B = 2 & \times 1 \end{cases}$$

$$\begin{cases} 2A + 2B = 2 \\ -2A - 18B = 2 \quad + \end{cases}$$

$-16B = 4$

$B = -\frac{1}{4}$ dus $A = 1\frac{1}{4}$

$u(t)=1\frac{1}{4}e^{-2t}-\frac{1}{4}e^{-18t}$

b. $D=\dfrac{k^2-4cm}{m^2}=\dfrac{40^2-4\cdot 72\cdot 8}{8^2}=-11<0$ dus onderdemping ofwel een gedempte trilling

c. $D\geq 0$

$D=\dfrac{k^2-4cm}{m^2}=\dfrac{1600-288m}{m^2}\geq 0$

de noemer is altijd positief, dus moet gelden:

$1600-288m\geq 0$

$-288m\geq -1600$

$m\leq 5\frac{5}{9}$

Opgave 69:

a. $F=-1000\frac{du}{dt}-500u$ en $\frac{d^2u}{dt^2}=\dfrac{F}{m}=\dfrac{F}{1000}$

$\frac{d^2u}{dt^2}=-\frac{du}{dt}-\frac{1}{2}u$

$\frac{d^2u}{dt^2}+\frac{du}{dt}+\frac{1}{2}u=0$

$\lambda^2+\lambda+\frac{1}{2}=0$

$(\lambda+\frac{1}{2})^2-\frac{1}{4}+\frac{1}{2}=0$

$(\lambda+\frac{1}{2})^2=-\frac{1}{4}$

$\lambda+\frac{1}{2}=\frac{1}{2}i \quad\vee\quad \lambda+\frac{1}{2}=-\frac{1}{2}i$

$\lambda=-\frac{1}{2}+\frac{1}{2}i \quad\vee\quad \lambda=-\frac{1}{2}-\frac{1}{2}i$

$u(t)=(A\cos(\frac{1}{2}t)+B\sin(\frac{1}{2}t))\cdot e^{-\frac{1}{2}t}$

$u(0)=A=1$

$u'(t)=(-\frac{1}{2}A\sin(\frac{1}{2}t)+\frac{1}{2}B\cos(\frac{1}{2}t))\cdot e^{-\frac{1}{2}t}+(A\cos(\frac{1}{2}t)+B\sin(\frac{1}{2}t))\cdot -\frac{1}{2}e^{-\frac{1}{2}t}$

$u'(0)=\frac{1}{2}B-\frac{1}{2}A=0$

$\frac{1}{2}B=\frac{1}{2}A$

$B=A=1$

$u(t)=(\cos(\frac{1}{2}t)+\sin(\frac{1}{2}t))\cdot e^{-\frac{1}{2}t}$

b. $D\geq 0$

$D=\dfrac{k^2-4cm}{m^2}=\dfrac{k^2-4\cdot 500\cdot 1000}{1000^2}=\dfrac{k^2-2000000}{1000000}\geq 0$

$k^2-2000000\geq 0$

$k^2\geq 2000000$

$k\leq -\sqrt{2000000}=-1000\sqrt{2} \quad\vee\quad k\geq 1000\sqrt{2}$

dus $k\geq 1000\sqrt{2}$

Opgave 70:

a. $F=0\cdot\frac{du}{dt}-c\cdot u$

$\frac{d^2u}{dt^2}=\dfrac{F}{m}=-\dfrac{c}{m}\cdot u$

$\frac{d^2u}{dt^2} + \frac{c}{m} \cdot u = 0$

$\lambda^2 + \frac{c}{m} = 0$

$\lambda^2 = -\frac{c}{m}$

$\lambda^2 = \frac{c}{m} \cdot i^2$

$\lambda = i \cdot \sqrt{\frac{c}{m}} \quad \vee \quad \lambda = -i \cdot \sqrt{\frac{c}{m}}$

$u(t) = A\cos\left(t \cdot \sqrt{\frac{c}{m}}\right) + B\sin(t \cdot \sqrt{\frac{c}{m}})$

$periode = \frac{2\pi}{\sqrt{\frac{c}{m}}} = 2\pi \cdot \sqrt{\frac{m}{c}}$

dus $T = 2\pi \cdot \sqrt{\frac{m}{c}}$

b. $F = -k \cdot \frac{du}{dt} - c \cdot u$

$\frac{d^2u}{dt^2} = \frac{F}{m} = -\frac{k}{m} \cdot \frac{du}{dt} - \frac{c}{m} \cdot u$

$\frac{d^2u}{dt^2} + \frac{k}{m} \cdot \frac{du}{dt} + \frac{c}{m} \cdot u = 0$

$\lambda^2 + \frac{k}{m} \cdot \lambda + \frac{c}{m} = 0$

$D \geq 0$ geeft $\left(\frac{k}{m}\right)^2 - 4 \cdot \frac{c}{m} \geq 0$

$(\frac{k}{m})^2 - 4c \cdot m \geq 0$

$(\frac{k}{m})^2 \geq 4c \cdot m$

$\frac{k}{m} \geq 2\sqrt{c \cdot m}$

<u>Opgave 71:</u>

a. $D = \frac{k^2 - 4cm}{m^2} = 0$

$k^2 - 4cm = 0$

$k^2 = 4cm$

$k = 2\sqrt{cm} = 2\sqrt{25000 \cdot 30} = 1732$

b. $F = -1730 \cdot \frac{du}{dt} - 25000u$

$\frac{d^2u}{dt^2} = \frac{F}{50}$

$\frac{d^2u}{dt^2} = -34{,}6\frac{du}{dt} - 500u$

$\frac{d^2u}{dt^2} + 34{,}6\frac{du}{dt} + 500u = 0$

$\lambda^2 + 34{,}6\lambda + 500 = 0$

$(\lambda + 17{,}3)^2 - 299{,}29 + 500 = 0$

$(\lambda + 17{,}3)^2 = -200{,}71$

$\lambda + 17{,}3 = i\sqrt{200{,}71} \quad \vee \quad \lambda + 17{,}3 = -i\sqrt{200{,}71}$

$\lambda = -17{,}3 + i\sqrt{200{,}71} \quad \vee \quad \lambda = -17{,}3 - i\sqrt{200{,}71}$

$\lambda = -17{,}3 + 14{,}2i \quad \vee \quad \lambda = -17{,}3 - 14{,}2i$

$u = (A\cos(14{,}2t) + B\sin(14{,}2t)) \cdot e^{-17{,}3t}$

dus de eigenfrequentie is: $\frac{14{,}2}{2\pi} = 2{,}3$ Hz

c. 70 omwentelingen per minuut is 1,17 omwentelingen per seconde
het veersysteem krijgt dus ongeveer 2,34 keer per seconde een duw naar beneden, waardoor resonantie kan ontstaan.

Opgave 72:

a. $D = \dfrac{k^2 - 4cm}{m^2} = \dfrac{k^2 - 4 \cdot 200000 \cdot 1000}{1000^2} = \dfrac{k^2 - 800000000}{1000000} = 0$

$k^2 - 800000000 = 0$

$k^2 = 800000000$

$k = \sqrt{800000000} \quad \vee \quad k = -\sqrt{800000000}$ (k.n.)

$k = 20000\sqrt{2}$

b. $F = -20000\sqrt{2} \cdot \frac{du}{dt} - 200000u$

$\frac{d^2u}{dt^2} = \dfrac{F}{1000}$

$\frac{d^2u}{dt^2} = -20\sqrt{2}\,\frac{du}{dt} - 200u$

$\frac{d^2u}{dt^2} + 20\sqrt{2}\,\frac{du}{dt} + 200u = 0$

$\lambda^2 + 20\sqrt{2}\lambda + 200 = 0$

$\lambda = \dfrac{-20\sqrt{2} \pm \sqrt{0}}{2} = -10\sqrt{2}$

$u(t) = (A + Bt) \cdot e^{-10\sqrt{2}t}$

$u(0) = A = 1{,}5$

$u' = B \cdot e^{-10\sqrt{2}t} + (A + Bt) \cdot e^{-10\sqrt{2}t} \cdot -10\sqrt{2}$

$u'(0) = B - 10\sqrt{2} \cdot A = 0$

$B = 10\sqrt{2} \cdot A = 10\sqrt{2} \cdot 1{,}5 = 15\sqrt{2}$

$u(t) = (1{,}5 + 15\sqrt{2} \cdot t) \cdot e^{-10\sqrt{2}t}$

c. $u(t) = (1{,}5 + 15\sqrt{2} \cdot t) \cdot e^{-10\sqrt{2}t} = 0{,}01$

neem $y_1 = (1{,}5 + 15\sqrt{2} \cdot x) \cdot e^{-10\sqrt{2}x}$ en $y_2 = 0{,}01$

intersect geeft $x = 0{,}502$

dus na ongeveer 0,5 seconde is de uitwijking minder dan 1 cm