

16.2 Limieten en asymptoten

Opgave 25:

- a. $f(-10) = 3,462$
 $f(-100) = 3,059$
 $f(-1000) = 3,006$
- b. door x heel erg negatief te worden daalt $f(x)$ steeds dichterbij 3
- c. $f(10) = 2,308$
 $f(100) = 2,939$
 $f(1000) = 2,994$
- d. door x heel erg positief te nemen stijgt $f(x)$ steeds dichterbij 3
- e. $y = 3$

Opgave 26:

- a. $\lim_{x \rightarrow \infty} \frac{3x^2 - x}{2 - x^2} = \lim_{x \rightarrow \infty} \frac{3 - \frac{1}{x}}{\frac{2}{x^2} - 1} = \frac{3 - 0}{0 - 1} = -3$
- b. $\lim_{x \rightarrow \infty} \frac{5 - x^3}{2x^3} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x^3} - 1}{2} = \frac{0 - 1}{2} = -\frac{1}{2}$
- c. $\lim_{x \rightarrow \infty} \frac{6x^2}{2 - x^3} = \lim_{x \rightarrow \infty} \frac{\frac{6}{x}}{\frac{2}{x^3} - 1} = \frac{0}{0 - 1} = 0$
- d. $\lim_{x \rightarrow \infty} \frac{(2x - 3)^2}{x^2 + 1} = \lim_{x \rightarrow \infty} \frac{4x^2 - 12x + 9}{x^2 + 1} = \lim_{x \rightarrow \infty} \frac{4 - \frac{12}{x} + \frac{9}{x^2}}{1 + \frac{1}{x^2}} = \frac{4 - 0 + 0}{1 + 0} = 4$

Opgave 27:

- a. $\lim_{x \rightarrow \infty} \frac{(4x - 1)^2}{x^3 + 4} = \lim_{x \rightarrow \infty} \frac{16x^2 - 8x + 1}{x^3 + 4} = \lim_{x \rightarrow \infty} \frac{\frac{16}{x} - \frac{8}{x^2} + \frac{1}{x^3}}{1 + \frac{4}{x^3}} = \frac{0 - 0 + 0}{1 + 0} = 0$
- b. $\lim_{x \rightarrow \infty} \frac{x^3 + 2}{(3x + 1)^2} = \lim_{x \rightarrow \infty} \frac{x^3 + 2}{9x^2 + 6x + 1} = \lim_{x \rightarrow \infty} \frac{x + \frac{2}{x^2}}{9 + \frac{6}{x} + \frac{1}{x^2}} = \text{bestaat niet}$
- c. $\lim_{x \rightarrow \infty} \frac{4x}{\sqrt{x^2 + 1}} = \lim_{x \rightarrow \infty} \frac{4x}{\sqrt{x^2(1 + \frac{1}{x^2})}} = \lim_{x \rightarrow \infty} \frac{4x}{-x\sqrt{1 + \frac{1}{x^2}}} = \lim_{x \rightarrow \infty} \frac{-4}{\sqrt{1 + \frac{1}{x^2}}} = \frac{-4}{\sqrt{1 + 0}} = -4$
- d. $\sqrt{2 - x}$ bestaat voor $x \leq 2$ dus $\lim_{x \rightarrow \infty} \frac{\sqrt{2 - x}}{\sqrt{x + 2}}$ bestaat niet

Opgave 28:

- a. $\lim_{x \rightarrow \infty} \frac{(2x + 1)^3}{x^3 + 1} = \lim_{x \rightarrow \infty} \frac{8x^3 + 12x^2 + 6x + 1}{x^3 + 1} = \lim_{x \rightarrow \infty} \frac{8 + \frac{12}{x} + \frac{6}{x^2} + \frac{1}{x^3}}{1 + \frac{1}{x^3}} = \frac{8 + 0 + 0 + 0}{1 + 0} = 8$
- b. $\lim_{x \rightarrow \infty} \frac{\sqrt{5x} + 50}{x - 1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} \cdot \sqrt{5x} + \frac{50}{x}}{1 - \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{5}}{x} + \frac{50}{x}}{1 - \frac{1}{x}} = \frac{0 + 0}{1 - 0} = 0$
- c. $\lim_{x \rightarrow \infty} \frac{2\sqrt{x^2 + 1}}{x + \sqrt{x}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x} \sqrt{x^2 + 1}}{1 + \frac{1}{x} \sqrt{x}} = \lim_{x \rightarrow \infty} \frac{2\sqrt{1 + \frac{1}{x^2}}}{1 + \sqrt{\frac{1}{x}}} = \frac{2\sqrt{1 + 0}}{1 + 0} = 2$

$$\begin{aligned}
 \text{d. } \lim_{x \rightarrow -\infty} \frac{3x^2 + x\sqrt{x^2 + 4}}{x^2 + 1} &= \lim_{x \rightarrow -\infty} \frac{3x^2 + \sqrt{x^2(1 + \frac{4}{x^2})}}{x^2 + 1} = \lim_{x \rightarrow -\infty} \frac{3x^2 - x^2\sqrt{1 + \frac{4}{x^2}}}{x^2 + 1} = \\
 &= \lim_{x \rightarrow -\infty} \frac{3 - \sqrt{1 + \frac{4}{x^2}}}{1 + \frac{1}{x^2}} = \frac{3 - \sqrt{1 + 0}}{1 + 0} = 2
 \end{aligned}$$

Opgave 29:

$$\lim_{x \rightarrow \infty} \frac{4x^2 - 1}{x^2 - x - 6} = \lim_{x \rightarrow \infty} \frac{4 - \frac{1}{x^2}}{1 - \frac{1}{x} - \frac{6}{x^2}} = \frac{4 - 0}{1 - 0 - 0} = 4 \text{ dus de H.A. is } y = 4$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$x = 3 \vee x = -2$ dus de V.A. zijn $x = 3$ en $x = -2$

Opgave 30:

$$\lim_{x \rightarrow \infty} \frac{6x}{\sqrt{2x^2 - 6}} = \lim_{x \rightarrow \infty} \frac{6x}{\sqrt{x^2(2 - \frac{6}{x^2})}} = \lim_{x \rightarrow \infty} \frac{6x}{x\sqrt{2 - \frac{6}{x^2}}} = \lim_{x \rightarrow \infty} \frac{6}{\sqrt{2 - \frac{6}{x^2}}} = \frac{6}{\sqrt{2 + 0}} = 3\sqrt{2}$$

$$\lim_{x \rightarrow -\infty} \frac{6x}{\sqrt{2x^2 - 6}} = \lim_{x \rightarrow -\infty} \frac{6x}{\sqrt{x^2(2 - \frac{6}{x^2})}} = \lim_{x \rightarrow -\infty} \frac{6x}{-x\sqrt{2 - \frac{6}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{6}{-\sqrt{2 - \frac{6}{x^2}}} = \frac{6}{-\sqrt{2 + 0}} = -3\sqrt{2}$$

dus de H.A. zijn $y = 3\sqrt{2}$ en $y = -3\sqrt{2}$

$$2x^2 - 6 = 0$$

$$2x^2 = 6$$

$$x^2 = 3$$

$x = \sqrt{3} \vee x = -\sqrt{3}$ dus de V.A. zijn $x = \sqrt{3}$ en $x = -\sqrt{3}$

Opgave 31:

a. $bx^2 - 18 = 0$ voor $x = 3$ en voor $x = -3$

$$9b - 18 = 0$$

$$9b = 18$$

$$b = 2$$

$$\lim_{x \rightarrow \infty} \frac{ax^2 + 5}{2x^2 - 18} = \lim_{x \rightarrow \infty} \frac{a + \frac{5}{x^2}}{2 - \frac{18}{x^2}} = \frac{a + 0}{2 - 0} = \frac{1}{2}a = 6$$

dus $a = 12$

b. $ax^4 + bx^3 - 2 = 0$ voor $x = -2$ en voor $x = 2$

$$16a - 8b - 2 = 0$$

$$16a + 8b - 2 = 0 \quad +$$

$$32a - 4 = 0$$

$$32a = 4$$

$$a = \frac{1}{8}$$

$$b = 0$$

Opgave 32:

- a. $\lim_{x \rightarrow \infty} f(x) = 0$
- b. $\lim_{x \rightarrow \infty} f(x) = \frac{a_n}{b_m}$
- c. $\lim_{x \rightarrow \infty} f(x) = \text{bestaat niet}$

Opgave 33:

a. $\frac{x^2 + 5x + 2}{x + 2} = \frac{x^2 + 5x + 6 - 4}{x + 2} = \frac{(x + 2)(x + 3) - 4}{x + 2} = x + 3 - \frac{4}{x + 2}$

b. $\lim_{x \rightarrow \infty} \frac{4}{x + 2} = \lim_{x \rightarrow \infty} \frac{\frac{4}{x}}{1 + \frac{2}{x}} = \frac{0}{1 + 0} = 0$

$\lim_{x \rightarrow -\infty} \frac{4}{x + 2} = \lim_{x \rightarrow -\infty} \frac{\frac{4}{x}}{1 + \frac{2}{x}} = \frac{0}{1 + 0} = 0$

dus de grafiek gaat op den duur steeds meer lijken op de lijn $y = x + 3$

Opgave 34:

a. $x - 1 = 0$

$x = 1$ dus de V.A. is $x = 1$

$f(x) = \frac{2x^2 + 4x - 5}{x - 1} = 2x + 6 + \frac{1}{x - 1}$ dus de S.A. is $y = 2x + 6$

b. $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$ dus de V.A. is $x = \frac{1}{2}$

$g(x) = \frac{-x^2 + 3x - 2}{2x - 1} = -\frac{1}{2}x + 1\frac{1}{4} - \frac{\frac{3}{4}}{2x - 1}$ dus de S.A. is $y = -\frac{1}{2}x + 1\frac{1}{4}$

c. $x^2 - 9 = 0$

$x^2 = 9$

$x = 3 \vee x = -3$ dus de V.A. zijn $x = 3$ en $x = -3$

$h(x) = \frac{x^3 + 3x^2 - 9x + 31}{x^2 - 9} = x + 3 + \frac{58}{x^2 - 9}$ dus de S.A. is $y = x + 3$

d. $x^2 - 1 = 0$

$x^2 = 1$

$x = 1 \vee x = -1$ dus de V.A. zijn $x = 1$ en $x = -1$

$j(x) = \frac{x^3 - 1}{x^2 - 1} = x + \frac{x - 1}{x^2 - 1}$ dus de S.A. is $y = x$

Opgave 35:

$f(x) = \frac{x^2 + 3x + 2}{|x|} = \frac{x^2 + 3x + 2}{x} = x + 3 + \frac{2}{x}$ als $x > 0$

$f(x) = \frac{x^2 + 3x + 2}{|x|} = \frac{x^2 + 3x + 2}{-x} = -x - 3 - \frac{2}{x}$ als $x < 0$

V.A.: $x = 0$

S.A.: $y = x + 3$ als $x \rightarrow \infty$
 $y = -x - 3$ als $x \rightarrow -\infty$

Opgave 36:

a. $f(x) = \frac{x^2 - x - 2}{x + 2} = \frac{(x + 2)(x - 3) + 4}{x + 2} = x - 3 + \frac{4}{x + 2}$

V.A.: $x = -2$

S.A.: $y = x - 3$

b. $f'(x) = \frac{(x + 2)(2x - 1) - (x^2 - x - 2)}{(x + 2)^2} = \frac{2x^2 + 3x - 2 - x^2 + x + 2}{(x + 2)^2} = \frac{x^2 + 4x}{(x + 2)^2} = 0$

$x^2 + 4x = 0$

$x(x + 4) = 0$

$x = 0 \vee x = -4$

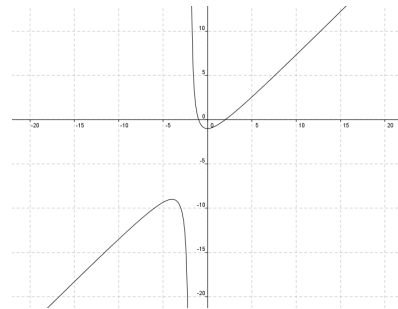
$\max f(-4) = -9$

$\min f(0) = -1$

c. $x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2 \vee x = -1$



$Opp(V) = -\int_{-1}^2 \frac{x^2 - x - 2}{x + 2} dx = -\int_{-1}^2 (x - 3 + \frac{4}{x + 2}) dx =$

$= -[\frac{1}{2}x^2 - 3x + 4 \ln|x + 2|]_{-1}^2$

$= -(2 - 6 + 4 \ln 4 - (\frac{1}{2} + 3 + 0)) = -(4 \ln 4 - 7\frac{1}{2}) = 7\frac{1}{2} - 4 \ln 4$

Opgave 37:

a. $f(10) = \sqrt{340} = 18,44$

$f(100) = \sqrt{39400} = 198,5$

$f(1000) = \sqrt{3994000} = 1998,5$

$g(10) = 20$

$g(100) = 200$

$g(1000) = 2000$

b. als $x \rightarrow \infty$ dan $f(x) - g(x) \rightarrow 0$

c. als $x \rightarrow -\infty$ dan $f(x) - h(x) \rightarrow 0$

Opgave 38:

$f(x) = \sqrt{4x^2 - 6x}$

$f'(x) = \frac{4x - 3}{\sqrt{4x^2 - 6x}}$

$a = \lim_{x \rightarrow \infty} \frac{4x - 3}{\sqrt{4x^2 - 6x}} = \lim_{x \rightarrow \infty} \frac{4x - 3}{\sqrt{x^2(4 - \frac{6}{x})}} = \lim_{x \rightarrow \infty} \frac{4x - 3}{x\sqrt{4 - \frac{6}{x}}} = \lim_{x \rightarrow \infty} \frac{4 - \frac{3}{x}}{\sqrt{4 - \frac{6}{x}}} = \frac{4 - 0}{\sqrt{4 - 0}} = \frac{4}{2} = 2$

$b = \lim_{x \rightarrow \infty} \sqrt{4x^2 - 6x} - 2x = \lim_{x \rightarrow \infty} (\sqrt{4x^2 - 6x} - 2x) \cdot \frac{\sqrt{4x^2 - 6x} + 2x}{\sqrt{4x^2 - 6x} + 2x} =$

$$\begin{aligned}
&= \lim_{x \rightarrow \infty} \frac{4x^2 - 6x - 4x^2}{\sqrt{4x^2 - 6x} + 2x} = \lim_{x \rightarrow \infty} \frac{-6x}{\sqrt{4x^2(1 - \frac{6}{4x})} + 2x} = \lim_{x \rightarrow \infty} \frac{-6x}{2x\sqrt{1 - \frac{3}{2x}} + 2x} = \\
&= \lim_{x \rightarrow \infty} \frac{-6}{2\sqrt{1 - \frac{3}{2x}} + 2} = \frac{-6}{2\sqrt{1 - 0} + 2} = -1\frac{1}{2}
\end{aligned}$$

dus de S.A. is $y = 2x - 1\frac{1}{2}$

Opgave 39:

$$f(x) = \sqrt{4x^2 + 6}$$

$$f'(x) = \frac{1}{2\sqrt{4x^2 + 6}} \cdot 8x = \frac{4x}{\sqrt{4x^2 + 6}}$$

$$a = \lim_{x \rightarrow \infty} \frac{4x}{\sqrt{4x^2 + 6}} = \lim_{x \rightarrow \infty} \frac{4x}{\sqrt{x^2(4 + \frac{6}{x^2})}} = \lim_{x \rightarrow \infty} \frac{4x}{x\sqrt{4 + \frac{6}{x^2}}} = \lim_{x \rightarrow \infty} \frac{4}{\sqrt{4 + \frac{6}{x^2}}} = \frac{4}{\sqrt{4 + 0}} = 2$$

$$\begin{aligned}
b &= \lim_{x \rightarrow \infty} \sqrt{4x^2 + 6} - 2x = \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 6} - 2x) \cdot \frac{\sqrt{4x^2 + 6} + 2x}{\sqrt{4x^2 + 6} + 2x} \\
&= \lim_{x \rightarrow \infty} \frac{4x^2 + 6 - 4x^2}{\sqrt{4x^2 + 6} + 2x} = \lim_{x \rightarrow \infty} \frac{6}{\sqrt{4x^2(1 + \frac{6}{4x^2})} + 2x} = \lim_{x \rightarrow \infty} \frac{6}{2x\sqrt{1 + \frac{6}{2x^2}} + 2x} \\
&= \lim_{x \rightarrow \infty} \frac{\frac{6}{x}}{2\sqrt{1 + \frac{6}{2x^2}} + 2} = \frac{0}{2\sqrt{1 + 0} + 2} = 0
\end{aligned}$$

Dus als $x \rightarrow \infty$ dan is de S.A.: $y = 2x$

$$a = \lim_{x \rightarrow -\infty} \frac{4x}{\sqrt{4x^2 + 6}} = \lim_{x \rightarrow -\infty} \frac{4x}{\sqrt{x^2(4 + \frac{6}{x^2})}} = \lim_{x \rightarrow -\infty} \frac{4x}{-x\sqrt{4 + \frac{6}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{4}{-\sqrt{4 + \frac{6}{x^2}}} = \frac{-4}{\sqrt{4 + 0}} = -2$$

$$\begin{aligned}
b &= \lim_{x \rightarrow -\infty} \sqrt{4x^2 + 6} + 2x = \lim_{x \rightarrow -\infty} (\sqrt{4x^2 + 6} + 2x) \cdot \frac{\sqrt{4x^2 + 6} - 2x}{\sqrt{4x^2 + 6} - 2x} \\
&= \lim_{x \rightarrow -\infty} \frac{4x^2 + 6 - 4x^2}{\sqrt{4x^2 + 6} - 2x} = \lim_{x \rightarrow -\infty} \frac{6}{\sqrt{4x^2(1 + \frac{6}{4x^2})} - 2x} = \lim_{x \rightarrow -\infty} \frac{6}{-2x\sqrt{1 + \frac{6}{2x^2}} - 2x} \\
&= \lim_{x \rightarrow -\infty} \frac{\frac{6}{x}}{-2\sqrt{1 + \frac{6}{2x^2}} - 2} = \frac{0}{-2\sqrt{1 + 0} - 2} = 0
\end{aligned}$$

Dus als $x \rightarrow -\infty$ dan is de S.A.: $y = -2x$

Opgave 40:

a. $f(x) = \sqrt{x^2 - 4x}$

$$x^2 - 4x \geq 0$$

$$x(x - 4) = 0$$

$$x = 0 \quad \vee \quad x = 4$$

$$\text{dus } x \leq 0 \quad \vee \quad x \geq 4$$

b. $f'(x) = \frac{1}{2\sqrt{x^2 - 4x}} \cdot (2x - 4) = \frac{x - 2}{\sqrt{x^2 - 4x}}$

$$a = \lim_{x \rightarrow \infty} \frac{x-2}{\sqrt{x^2-4x}} = \lim_{x \rightarrow \infty} \frac{x-2}{\sqrt{x^2(1-\frac{4}{x})}} = \lim_{x \rightarrow \infty} \frac{x-2}{x\sqrt{1-\frac{4}{x}}} = \lim_{x \rightarrow \infty} \frac{1-\frac{2}{x}}{\sqrt{1-\frac{4}{x}}} = \frac{1-0}{\sqrt{1-0}} = 1$$

$$\begin{aligned} b &= \lim_{x \rightarrow \infty} \sqrt{x^2-4x} - x = \lim_{x \rightarrow \infty} (\sqrt{x^2-4x} - x) \cdot \frac{\sqrt{x^2-4x} + x}{\sqrt{x^2-4x} + x} \\ &= \lim_{x \rightarrow \infty} \frac{x^2-4x-x^2}{\sqrt{x^2-4x} + x} = \lim_{x \rightarrow \infty} \frac{-4x}{\sqrt{x^2(1-\frac{4}{x})} + x} = \lim_{x \rightarrow \infty} \frac{-4x}{x\sqrt{1-\frac{4}{x}} + x} \\ &= \lim_{x \rightarrow \infty} \frac{-4}{\sqrt{1-\frac{4}{x}} + 1} = \frac{-4}{\sqrt{1-0} + 1} = -2 \end{aligned}$$

dus voor $x \rightarrow \infty$ is de S.A. $y = x - 2$

$$a = \lim_{x \rightarrow -\infty} \frac{x-2}{\sqrt{x^2-4x}} = \lim_{x \rightarrow -\infty} \frac{x-2}{\sqrt{x^2(1-\frac{4}{x})}} = \lim_{x \rightarrow -\infty} \frac{x-2}{-x\sqrt{1-\frac{4}{x}}} = \lim_{x \rightarrow -\infty} \frac{1-\frac{2}{x}}{-\sqrt{1-\frac{4}{x}}} = \frac{1-0}{-\sqrt{1-0}} = -1$$

$$\begin{aligned} b &= \lim_{x \rightarrow -\infty} \sqrt{x^2-4x} + x = \lim_{x \rightarrow -\infty} (\sqrt{x^2-4x} + x) \cdot \frac{\sqrt{x^2-4x} - x}{\sqrt{x^2-4x} - x} \\ &= \lim_{x \rightarrow -\infty} \frac{x^2-4x-x^2}{\sqrt{x^2-4x} - x} = \lim_{x \rightarrow -\infty} \frac{-4x}{\sqrt{x^2(1-\frac{4}{x})} - x} = \lim_{x \rightarrow -\infty} \frac{-4x}{-x\sqrt{1-\frac{4}{x}} - x} \\ &= \lim_{x \rightarrow -\infty} \frac{-4}{-\sqrt{1-\frac{4}{x}} - 1} = \frac{-4}{-\sqrt{1-0} - 1} = 2 \end{aligned}$$

dus voor $x \rightarrow -\infty$ is de S.A. $y = -x + 2$

c. $\sqrt{x^2-4x} < 2$

$$x^2 - 4x = 4$$

$$x^2 - 4x - 4 = 0$$

$$x = \frac{4 \pm \sqrt{32}}{2} = \frac{4 \pm 4\sqrt{2}}{2} = 2 \pm 2\sqrt{2}$$

$$2 - 2\sqrt{2} < x \leq 0 \quad \vee \quad 4 \leq x < 2 + 2\sqrt{2}$$

d. $\sqrt{x^2-4x} \leq x$

$$x^2 - 4x = x^2$$

$$-4x = 0$$

$$x = 0$$

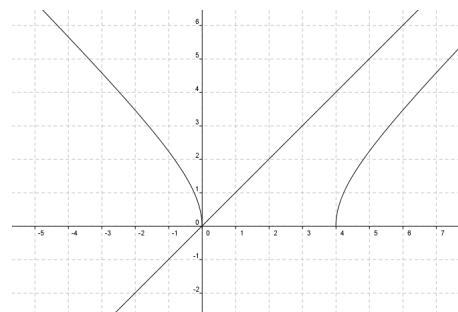
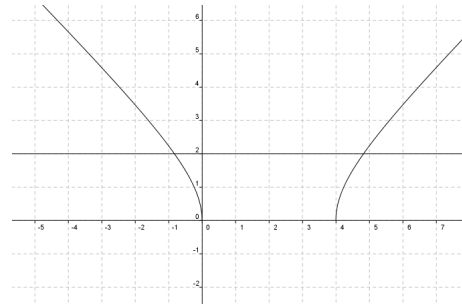
$$x = 0 \quad \vee \quad x \geq 4$$

e. $Inh = \pi \int_4^p (\sqrt{x^2-4x})^2 dx$

$$= \pi \int_4^p (x^2 - 4x) dx = \pi \left[\frac{1}{3} x^3 - 2x^2 \right]_4^p = \pi \left[\frac{1}{3} p^3 - 2p^2 - (21\frac{1}{3} - 32) \right] = 100\pi$$

$$\frac{1}{3} p^3 - 2p^2 + 10\frac{1}{3} = 100$$

Deze vergelijking is niet met de hand op te lossen, wel met de GR.



Opgave 41:

a. $f(x) = \frac{1}{2}x - 2 + \frac{2}{x+1}$

$$f'(x) = \frac{1}{2} - \frac{2}{(x+1)^2} = 0$$

$$\frac{2}{(x+1)^2} = \frac{1}{2}$$

$$(x+1)^2 = 4$$

$$x+1 = 2 \quad \vee \quad x+1 = -2$$

$$x = 1 \quad \vee \quad x = -3$$

$$\max f(-3) = -4\frac{1}{2}$$

$$\min f(1) = -\frac{1}{2}$$

b. V.A.: $x = -1$

S.A.: $y = \frac{1}{2}x - 2$

c. $\frac{1}{2}x - 2 + \frac{2}{x+1} = 0$

$$\left(\frac{1}{2}x - 2\right)(x+1) + 2 = 0$$

$$\frac{1}{2}x^2 - 1\frac{1}{2}x - 2 + 2 = 0$$

$$\frac{1}{2}x(x-3) = 0$$

$$x = 0 \quad \vee \quad x = 3$$

$$\begin{aligned} Opp(V) &= -\int_0^3 \left(\frac{1}{2}x - 2 + \frac{2}{x+1}\right) dx = -\left[\frac{1}{4}x^2 - 2x + 2\ln|x+1|\right]_0^3 = -(2\frac{1}{4} - 6 + 2\ln 4) \\ &= 3\frac{3}{4} - 2\ln 4 \end{aligned}$$

d. f moet continu zijn in $x = -3$ dus:

$$\lim_{x \uparrow -3} f(x) = f(-3) = -4\frac{1}{2}$$

$$\lim_{x \downarrow -3} ax^3 + bx^2 + cx + d = -27a + 9b - 3c + d$$

$$\text{dus } -27a + 9b - 3c + d = -4\frac{1}{2}$$

f moet differentieerbaar zijn in $x = -3$ dus moet ook nog gelden:

$$\lim_{x \uparrow -3} f'(x) = \lim_{x \uparrow -3} \frac{1}{2} - \frac{2}{(x+1)^2} = \frac{1}{2} - \frac{1}{2} = 0$$

$$\lim_{x \downarrow -3} f'(x) = \lim_{x \downarrow -3} 3ax^2 + 2bx + c = 27a - 6b + c$$

$$\text{dus } 27a - 6b + c = 0$$

f moet continu zijn in $x = 0$ dus:

$$\lim_{x \uparrow 0} ax^3 + bx^2 + cx + d = d$$

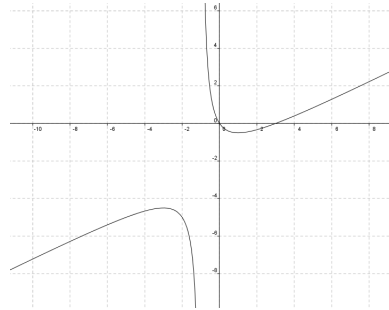
$$\lim_{x \downarrow 0} f(x) = f(0) = 0$$

$$\text{dus } d = 0$$

f moet differentieerbaar zijn in $x = 0$ dus moet ook nog gelden:

$$\lim_{x \uparrow 0} f'(x) = \lim_{x \uparrow 0} 3ax^2 + 2bx + c = c$$

$$\lim_{x \downarrow 0} f'(x) = \lim_{x \downarrow 0} \frac{1}{2} - \frac{2}{(x+1)^2} = \frac{1}{2} - 2 = -1\frac{1}{2}$$



$$\text{dus } c = -1\frac{1}{2}$$

$$\begin{cases} -27a + 9b + 4\frac{1}{2} = -4\frac{1}{2} \\ 27a - 6b - 1\frac{1}{2} = 0 \end{cases} +$$

$$3b + 3 = -4\frac{1}{2}$$

$$3b = -7\frac{1}{2}$$

$$b = -2\frac{1}{2}$$

$$27a - 6 \cdot -2\frac{1}{2} - 1\frac{1}{2} = 0$$

$$27a = -13\frac{1}{2}$$

$$a = -\frac{1}{2}$$