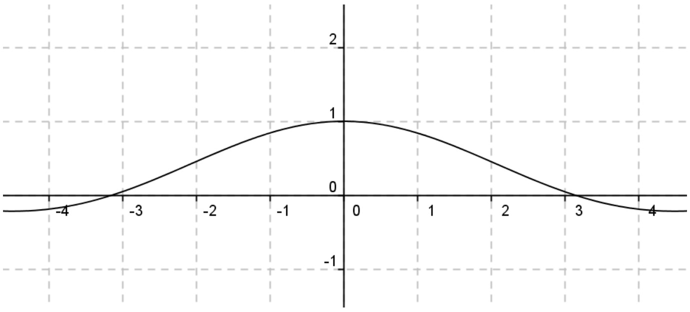
## 16.3 Standaardlimieten

**Opgave 42:**

a.




b. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

**Opgave 43:**

a. $\sin \alpha = \frac{d(B,OA)}{1} = d(B,OA)$

$\tan \alpha = \frac{AC}{OA} = AC$

oppervlakte sector $OAB = \frac{\alpha}{2\pi} \cdot \pi r^2 = \frac{1}{2}\alpha \cdot 1^2 = \frac{1}{2}\alpha$

$Opp(\Delta OAB) = \frac{1}{2} \cdot OA \cdot d(B,OA) = \frac{1}{2} \cdot 1 \cdot \sin \alpha = \frac{1}{2}\sin \alpha$

$Opp(\Delta OAC) = \frac{1}{2} \cdot OA \cdot AC = \frac{1}{2} \cdot 1 \cdot \tan \alpha = \frac{1}{2}\tan \alpha$

b. $Opp(\Delta OAB) \leq opp(\text{sec}tor\ OAB) \leq Opp(\Delta OAC)$

$\frac{1}{2}\sin \alpha \leq \frac{1}{2}\alpha \leq \frac{1}{2}\tan \alpha$

$\sin \alpha \leq \alpha \leq \tan \alpha$

**Opgave 44:**

a. $\lim_{x \uparrow 0} \frac{\sin x}{x} = \lim_{-x \downarrow 0} \frac{\sin x}{x} = \lim_{y \downarrow 0} \frac{\sin(-y)}{-y} = \lim_{y \downarrow 0} \frac{-\sin y}{-y} = \lim_{y \downarrow 0} \frac{\sin y}{y} = 1$

b. $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \cos x = 1 \cdot 1 = 1$

**Opgave 45:**

a. $\lim_{x \to 0} \frac{\sin 2x}{x} = \lim_{x \to 0} \frac{\sin 2x}{2x} \cdot 2 = 1 \cdot 2 = 2$

b. $\lim_{x \to 0} \frac{x}{\sin 3x} = \lim_{x \to 0} \frac{3x}{\sin 3x} \cdot \frac{1}{3} = 1 \cdot \frac{1}{3} = \frac{1}{3}$

c. $\lim_{x \to 0} \frac{x^2 + \tan x}{x} = \lim_{x \to 0} (x + \frac{\tan x}{x}) = 0 + 1 = 1$

d. $\lim_{x \to 0} \frac{\sin 4x}{\tan \frac{1}{2}x} = \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot \frac{\frac{1}{2}x}{\tan \frac{1}{2}x} \cdot \frac{4}{\frac{1}{2}} = 1 \cdot 1 \cdot 8 = 8$

e. $\lim_{x \to 0} \frac{2x + \sin x}{3x + \tan x} = \lim_{x \to 0} = \lim_{x \to 0} \frac{2 + \frac{\sin x}{x}}{3 + \frac{\tan x}{x}} = \frac{2+1}{3+1} = \frac{3}{4}$

f. $\lim\limits_{x\to\pi} \dfrac{2x\sin x}{\tan x} = \lim\limits_{x\to\pi} \dfrac{2x\sin x}{\frac{\sin x}{\cos x}} = \lim\limits_{x\to\pi} 2x\cos x = 2\pi \cdot -1 = -2\pi$

**<u>Opgave 46:</u>**

a. $\lim\limits_{x\to\frac{1}{2}\pi} \dfrac{\sin(x-\frac{1}{2}\pi)}{x-\frac{1}{2}\pi} = \lim\limits_{y\to 0} \dfrac{\sin y}{y} = 1$

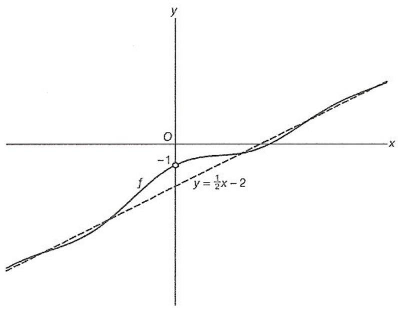
b. $\lim\limits_{x\to\frac{1}{2}\pi} \dfrac{\tan(x-\frac{1}{2}\pi)}{\sin(\frac{1}{2}x-\frac{1}{4}\pi)} = \lim\limits_{x\to\frac{1}{2}\pi} \dfrac{\tan(x-\frac{1}{2}\pi)}{\sin(\frac{1}{2}(x-\frac{1}{2}\pi))} = \lim\limits_{y\to 0} \dfrac{\tan y}{\sin\frac{1}{2}y} = \lim\limits_{y\to 0} \dfrac{\tan y}{y} \cdot \dfrac{\frac{1}{2}y}{\sin\frac{1}{2}y} \cdot 2 = 1 \cdot 1 \cdot 2 = 2$

c. $\lim\limits_{x\to 0} \dfrac{x^2+\sin 2x}{2x+\sin x} = \lim\limits_{x\to 0} \dfrac{2x+2\cos 2x}{2+\cos x} = \dfrac{0+2}{2+1} = \frac{2}{3}$ (gebruik de l'Hospital)

d. $\lim\limits_{x\downarrow 0} \dfrac{x\sqrt{x}+\tan x}{x+\sin 3x} = \lim\limits_{x\downarrow 0} \dfrac{1\frac{1}{2}\sqrt{x}+\frac{1}{\cos^w x}}{1+3\cos 3x} = \dfrac{0+1}{1+3} = \frac{1}{4}$ (gebruik de l'Hospital)

**<u>Opgave 47:</u>**

a. $\lim\limits_{x\to 0} \dfrac{x^2-4x+2\sin x}{2x} = \lim\limits_{x\to 0}(\frac{1}{2}x-2+\dfrac{\sin x}{x}) = 0-2+1=-1$




als $x\to\infty$ dan $f(x)\to\infty$

als $x\to-\infty$ dan $f(x)\to-\infty$

$B_f = \langle\leftarrow,-1\rangle \cup \langle -1,\rightarrow\rangle$

b. $y=\frac{1}{2}x-2$

c. $y_1 = \dfrac{x^2-4x+2\sin x}{2x}$ optie zero geeft: $x = 4{,}43$

dus $x \le 4{,}43$

**<u>Opgave 48:</u>**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} =$$

$$= \lim_{h\to 0} \frac{2\sin(\frac{1}{2}(x+h-x))\cdot\cos(\frac{1}{2}(x+h+x))}{h}$$

$$= \lim_{h\to 0} \frac{2\sin\frac{1}{2}h\cdot\cos(x+\frac{1}{2}h)}{h}$$

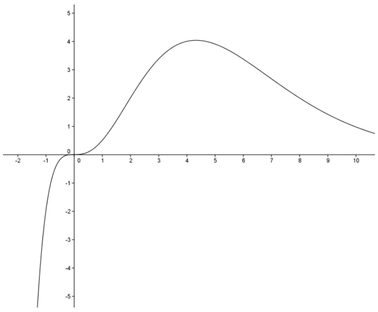
$$= \lim_{h\to 0} \frac{\sin\frac{1}{2}h\cdot\cos(x+\frac{1}{2}h)}{\frac{1}{2}h}$$

$$= \lim_{h\to 0} \frac{\sin\frac{1}{2}h}{\frac{1}{2}h}\cdot\cos(x+\frac{1}{2}h)$$

$$= 1\cdot\cos x = \cos x$$

## Opgave 49:

a.



b. $\lim\limits_{x\to\infty} \dfrac{x^3}{2^x} = 0$

c. $f(x) = \dfrac{x^3}{2^x} = x^3 \cdot 2^{-x} = 2^{\,{}^2\log x^3} \cdot 2^{-x} = 2^{3 \cdot {}^2\log x} \cdot 2^{-x} = 2^{3 \cdot {}^2\log x - x}$

## Opgave 50:

a. $\lim\limits_{x\to\infty} \dfrac{\ln x}{x} = \lim\limits_{t\to\infty} \dfrac{\ln e^{\frac{t}{n}}}{e^t} = \lim\limits_{t\to\infty} \dfrac{\frac{t}{n}}{e^t} = \lim\limits_{t\to\infty} \dfrac{1}{n} \cdot \dfrac{t}{e^t} = \dfrac{1}{n} \cdot \lim\limits_{t\to\infty} \dfrac{t}{e^t} = \dfrac{1}{n} \cdot 0 = 0$

b. $\lim\limits_{x\downarrow 0} (x^n \cdot \ln x) = \lim\limits_{t\to\infty} (\tfrac{1}{t})^n \cdot \ln \tfrac{1}{t} = \lim\limits_{t\to\infty} \dfrac{\ln t^{-1}}{t^n} = \lim\limits_{t\to\infty} \dfrac{-\ln t}{t^n} = 0$

## Opgave 51:

a. $\lim\limits_{x\to\infty} \dfrac{x\sqrt{x}}{e^{2x}} = \lim\limits_{x\to\infty} \dfrac{x^{1\frac{1}{2}}}{(e^2)^x} = 0$

b. $\lim\limits_{x\to-\infty} x \cdot e^x = \lim\limits_{t\to\infty} -t \cdot e^{-t} = \lim\limits_{t\to\infty} -\dfrac{t}{e^t} = 0$

c. $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^2\sqrt{x}} = \lim\limits_{x\to\infty} \dfrac{\ln x}{x^{2\frac{1}{2}}} = 0$

d. $\lim\limits_{x\to\infty} \dfrac{\ln^3 x}{\sqrt[3]{x}} = \lim\limits_{x\to\infty} \dfrac{\ln^3 x}{x^{\frac{1}{3}}} = \lim\limits_{x\to\infty} \left( \dfrac{\ln x}{x^{\frac{1}{3}}} \right)^3 = 0^3 = 0$

e. $\lim\limits_{x\downarrow 0} \sqrt[3]{x} \cdot \ln x = \lim\limits_{x\downarrow 0} x^{\frac{1}{3}} \cdot \ln x = 0$

f. $\lim\limits_{x\downarrow 0} \sqrt{x} \cdot \ln^3 x = \lim\limits_{x\downarrow 0} x^{\frac{1}{2}} \cdot \ln^3 x = \lim\limits_{x\downarrow 0} (x^{\frac{1}{6}} \cdot \ln x)^3 = 0^3 = 0$

## Opgave 52:

a. $\lim\limits_{x\to\infty} \dfrac{x^2 + 4x}{e^x} = \lim\limits_{x\to\infty} \left( \dfrac{x^2}{e^x} + 4 \cdot \dfrac{x}{e^x} \right) = 0 + 4 \cdot 0 = 0$

b. $\lim\limits_{x\to-\infty} x^4 \cdot \sqrt{e^x} = \lim\limits_{t\to\infty} t^4 \cdot e^{-\frac{1}{2}t} = \lim\limits_{t\to\infty} \dfrac{t^4}{e^{\frac{1}{2}t}} = \lim\limits_{t\to\infty} \dfrac{t^4}{(e^{\frac{1}{2}})^t} = 0$

c. $\lim\limits_{x\to\infty} \dfrac{\sqrt[3]{x}}{x^e} = \lim\limits_{x\to\infty} \dfrac{\ln^{\frac{1}{3}} x}{x^e} = \lim\limits_{x\to\infty} \left( \dfrac{\ln x}{x^{3e}} \right)^{\frac{1}{3}} = 0^{\frac{1}{3}} = 0$

d. $\lim\limits_{x\to\infty}\left(\frac{10}{x}\right)^e \cdot \sqrt{\ln x} = \lim\limits_{x\to\infty}\frac{10^e}{x^e}\cdot \ln^{\frac{1}{2}} x = 10^e \cdot \lim\limits_{x\to\infty}\left(\frac{\ln x}{x^{2e}}\right)^{\frac{1}{2}} = 10^e \cdot 0 = 0$

e. $\lim\limits_{x\downarrow 0}(x^2 + x)\ln x = \lim\limits_{x\downarrow 0}(x^2 \ln x + x\ln x) = 0 + 0 = 0$

f. $\lim\limits_{x\downarrow 0} x^2 e^x \ln^3 x = \lim\limits_{x\downarrow 0} e^x \cdot (x^{\frac{2}{3}} \ln x)^3 = e^0 \cdot 0^3 = 1\cdot 0 = 0$

**Opgave 53:**

a. $\lim\limits_{x\downarrow 0} x^x = \lim\limits_{x\downarrow 0} e^{\ln x^x} = \lim\limits_{x\downarrow 0} e^{x\cdot \ln x} = e^0 = 1$

b. $\lim\limits_{x\downarrow 0} x^{\frac{1}{x}} = \lim\limits_{x\downarrow 0} e^{\ln x^{\frac{1}{x}}} = \lim\limits_{x\downarrow 0} e^{\frac{1}{x}\cdot \ln x} = 0 \qquad (e^{-\infty} = 0)$

c. $\lim\limits_{x\to\infty} x^{\frac{1}{x}} = \lim\limits_{x\to\infty} e^{\ln x^{\frac{1}{x}}} = \lim\limits_{x\to\infty} e^{\frac{\ln x}{x}} = e^0 = 1$

**Opgave 54:**

a. $f(x) = ex \cdot \ln^2 x$

$f'(x) = e\cdot \ln^2 x + ex \cdot 2\ln x \cdot \frac{1}{x} = e\cdot \ln^2 x + 2e\cdot \ln x = 0$

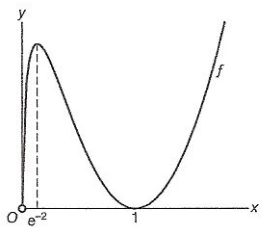
$e \cdot \ln x \cdot (\ln x + 2) = 0$

$\ln x = 0 \quad \vee \quad \ln x = -2$

$x = e^0 = 1 \quad \vee \quad x = e^{-2} = \frac{1}{e^2}$

max $f(\frac{1}{e^2}) = \frac{4}{e}$

min $f(1) = 0$




b. $\lim\limits_{x\downarrow 0} ex \cdot \ln^2 x = e \cdot \lim\limits_{x\downarrow 0}(x^{\frac{1}{2}} \ln x)^2 = e \cdot 0^2 = 0$

c. $y_1 = ex \cdot \ln^2 x$ en $y_2 = \ln x$

intersect geeft $x = 1 \quad \vee \quad x = 1{,}32$

$0 < x < 1 \quad \vee \quad x > 1{,}32$

**Opgave 55:**

a. $f(x) = x^{\frac{1}{x}} = e^{\ln x^{\frac{1}{x}}} == e^{\frac{1}{x}\ln x}$ f

$f'(x) = e^{\frac{1}{x}\ln x} \cdot (\frac{-1}{x^2}\ln x + \frac{1}{x}\cdot\frac{1}{x}) = e^{\frac{1}{x}\ln x} \cdot \frac{1}{x^2}\cdot(-\ln x + 1) = 0$

$e^{\frac{1}{x}\ln x} = 0 \quad \vee \quad \frac{1}{x^2} = 0 \quad \vee \quad -\ln x + 1 = 0$

$-\ln x = -1$

$\ln x = 1$

$x = e$

max $f(e) = e^{\frac{1}{e}}$

b. $\lim\limits_{x\downarrow 0} f(x) = \lim\limits_{x\downarrow 0} x^{\frac{1}{x}} = \lim\limits_{x\downarrow 0}(e^{\ln x})^{\frac{1}{x}} = \lim\limits_{x\downarrow 0} e^{\frac{\ln x}{x}} = 0$

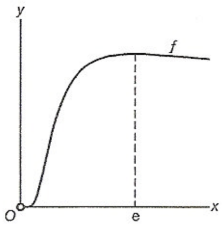
$f'(x) = \frac{1}{x^2}\cdot e^{\frac{1}{x}\ln x}\cdot(1 - \ln x) = \frac{x^{\frac{1}{x}}}{x^2}(1-\ln x) = x^{\frac{1}{x}-2}\cdot(1-\ln x) = x^{\frac{1}{x}-2} - x^{\frac{1}{x}-2}\cdot \ln x$

neem $p = \frac{1}{x} - 2$ dan is $\frac{1}{x} = p + 2$ dus $x = \frac{1}{p+2}$

$\lim_{x\downarrow 0} x^{\frac{1}{x}-2} = \lim_{p\to\infty}\left(\frac{1}{p+2}\right)^p = 0$

$\lim_{x\downarrow 0} x^{\frac{1}{x}-2}\cdot \ln x = 0$ (standaardlimiet)

dus $\lim_{x\downarrow 0} f'(x) = 0-0=0$




c. $\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} e^{\frac{\ln x}{x}} = e^0 = 1$ dus de H.A. is $y=1$

d. $f(x) = g(x)$

$x^{\frac{1}{x}} = x^x$

$\frac{1}{x} = x$

$x^2 = 1$

$x = 1 \quad \vee \quad x = -1$ (vervalt)

$f'(1) = \frac{1}{1^2}\cdot e^{\frac{1}{1}\cdot \ln 1}\cdot(1-\ln 1) = 1$

$g'(1) = 1^1 = 1$

$f(1) = g(1) \quad \wedge \quad f'(1) = g'(1)$ dus de grafieken van *f* en *g* raken elkaar in $x=1$

**Opgave 56:**

a. $f(x) = e^x$

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$

b. $\lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \lim_{x\to 0}\frac{e^x(e^h-1)}{h} = e^x\cdot\lim_{h\to 0}\frac{e^h-1}{h}$

$f'(x) = e^x$ dus $\lim_{h\to 0}\frac{e^x-1}{h} = 1$

**Opgave 57:**

$\lim_{x\to-\infty}\left(1+\frac{a}{x}\right)^x = \lim_{t\to\infty}\left(1+\frac{a}{-t}\right)^{-t} = \lim_{t\to\infty}\frac{1}{\left(1+\frac{-a}{t}\right)^t} = \frac{1}{e^{-a}} = e^a$

**Opgave 58:**

a. $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x = \lim_{x\to\infty}\left(1+\frac{-2}{x}\right)^x = e^{-2} = \frac{1}{e^2}$

b. $\lim_{x\to\infty}\left(\frac{x+4}{x}\right)^x = \lim_{x\to\infty}\left(1+\frac{4}{x}\right)^x = e^4$

c. $\lim_{x\to\infty}\left(\frac{x}{x+5}\right)^x = \lim_{x\to\infty}\left(\frac{1}{1+\frac{5}{x}}\right)^x = \lim_{x\to\infty}\frac{1^x}{\left(1+\frac{5}{x}\right)^x} = \frac{1}{e^5}$

d. $\lim_{x\to-\infty}\left(\frac{2x+1}{2x}\right)^x = \lim_{x\to-\infty}\left(1+\frac{1}{2x}\right)^x = \lim_{x\to\infty}\left(1+\frac{\frac{1}{2}}{x}\right)^x = e^{\frac{1}{2}} = \sqrt{e}$

**Opgave 59:**

a. stel $x^2 = t$

$\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^{x^2} = \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t = e^1 = e$

b. $\lim_{x\to\infty}\left(\frac{x^2+4}{x^2}\right)^{x^2} = \lim_{x\to\infty}\left(1+\frac{4}{x^2}\right)^{x^2} = e^4$

$$\lim_{x\to\infty}\left(\tfrac{x^2+4}{x^2}\right)^x = \lim_{x\to\infty}\left(1+\tfrac{4}{x^2}\right)^x = \lim_{x\to\infty}\left(\left(1+\tfrac{4}{x^2}\right)^{x^2}\right)^{\frac{1}{x}} = (e^4)^0 = 1$$