16.4 Limieten bij rijen

Opgave 60:

a. $S_{10} = \frac{1}{2} \cdot 11 \cdot (u_0 + u_{10}) = \frac{1}{2} \cdot 11 \cdot (5 + 35) = 220$

b. $S_{19} = \frac{1}{2} \cdot 20 \cdot (u_0 + u_{19}) = \frac{1}{2} \cdot 20 \cdot (5 + 65) = 670$

c. $S_n = \frac{1}{2}(n+1)(u_0 + u_n) = \frac{1}{2}(n+1)(5+5+3n)$

$= (\frac{1}{2}n + \frac{1}{2})(10+3n)$

$= 5n + 1\frac{1}{2}n^2 + 5 + 1\frac{1}{2}n$

$= 1\frac{1}{2}n^2 + 6\frac{1}{2}n + 5$

Opgave 61:

a. $S_{10} = \dfrac{u_0 - u_{11}}{1-r} = \dfrac{20 - 20 \cdot (\frac{3}{2})^{11}}{1 - \frac{3}{2}} = 3419{,}9$

b. $S_{19} = \dfrac{u_0 - u_{20}}{1-r} = \dfrac{20 - 20 \cdot (\frac{3}{2})^{20}}{1 - \frac{3}{2}} = 132970$

c. $S_n = \dfrac{u_0 - u_{n+1}}{1-r} = \dfrac{20 - 20 \cdot (\frac{3}{2})^{n+1}}{1 - \frac{3}{2}} = -40 + 40 \cdot (\frac{3}{2})^{n+1} = -40 + 40 \cdot \frac{3}{2} \cdot (\frac{3}{2})^n = 60 \cdot (\frac{3}{2})^n - 40$

Opgave 62:

a. $S_{10} = \dfrac{u_0 - u_{11}}{1-r} = \dfrac{60 - 60 \cdot (\frac{2}{3})^{11}}{1 - \frac{2}{3}} = 177{,}9$

b. $S_{19} = \dfrac{u_0 - u_{20}}{1-r} = \dfrac{60 - 60 \cdot (\frac{2}{3})^{20}}{1 - \frac{2}{3}} = 179{,}95$

c. $\lim\limits_{n\to\infty} u_n = \lim\limits_{n\to\infty} 60 \cdot (\frac{2}{3})^n = 0$

$\lim\limits_{n\to\infty} \sum\limits_{k=0}^{n} u_k = \lim\limits_{n\to\infty} \dfrac{u_0 - u_{n+1}}{1-r} = \lim\limits_{n\to\infty} \dfrac{60 - 60 \cdot (\frac{2}{3})^{n+1}}{1 - \frac{2}{3}} = \dfrac{60 - 0}{\frac{1}{3}} = 180$

Opgave 63:

a. $2 + 5 + 8 + \ldots + (3n+2) = \frac{1}{2} \cdot (n+1) \cdot (2 + 3n + 2)$

$= \frac{1}{2}(n+1)(3n+4)$

$= \frac{1}{2}(3n^2 + 7n + 4)$

$= 1\frac{1}{2}n^2 + 3\frac{1}{2}n + 2$

$\lim\limits_{n\to\infty} \dfrac{2 + 5 + 8 + \ldots + (3n+2)}{n^2} = \lim\limits_{n\to\infty} \dfrac{1\frac{1}{2}n^2 + 3\frac{1}{2}n + 2}{n^2} = \lim\limits_{n\to\infty} \dfrac{1\frac{1}{2} + \frac{3\frac{1}{2}}{n} + \frac{2}{n^2}}{1} = \dfrac{1\frac{1}{2} + 0 + 0}{1} = 1\frac{1}{2}$

b. $1 + 2 + 3 + \ldots + n = \frac{1}{2}n(1+n) = \frac{1}{2}n^2 + \frac{1}{2}n$

$\lim\limits_{n\to\infty} \dfrac{1 + 2 + 3 + \ldots + n}{n^2} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{2}n^2 + \frac{1}{2}n}{n^2} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{2} + \frac{1}{2n}}{1} = \dfrac{\frac{1}{2} + 0}{1} = \frac{1}{2}$

c. $1 + 2 + 4 + 8 + \ldots + 2^n = \dfrac{1 - 2^{n-1}}{1-2} = 2^{n+1} - 1$

$$\lim_{n\to\infty}\frac{1+2+4+8+...+2^n}{2^n}=\lim_{n\to\infty}\frac{2^{n+1}-1}{2^n}=\lim_{n\to\infty}(2-2^{-n})=2-0=2$$

d. $10+20+40+...+5\cdot2^n=\dfrac{10-5\cdot2^{n+1}}{1-2}=5\cdot2^{n+1}-10$

$$\lim_{n\to\infty}\frac{5\cdot2^n}{10+20+40+...+5\cdot2^n}=\lim_{n\to\infty}\frac{5\cdot2^n}{5\cdot2^{n+1}-10}=\lim_{n\to\infty}\frac{5}{10-\frac{10}{2^n}}=\frac{5}{10-0}=\tfrac{1}{2}$$

Opgave 64:

a. $1+5+9+13+...+4n+1=\tfrac{1}{2}(n+1)(1+4n+1)=(\tfrac{1}{2}n+\tfrac{1}{2})(4n+2)=2n^2+3n+1$

$1+4+7+10+...+3n+1=\tfrac{1}{2}(n+1)(1+3n+1)=(\tfrac{1}{2}+\tfrac{1}{2}n)(3n+2)=1\tfrac{1}{2}n^2+2\tfrac{1}{2}n+1$

$$\lim_{n\to\infty}\frac{1+5+9+13+...+4n+1}{1+4+7+10+...+3n+1}=\lim_{n\to\infty}\frac{2n^2+3n+1}{1\tfrac{1}{2}n^2+2\tfrac{1}{2}n+1}=\lim_{n\to\infty}\frac{2+\frac{3}{n}+\frac{1}{n^2}}{1\tfrac{1}{2}+\frac{2\frac{1}{2}}{n}+\frac{1}{n^2}}=\frac{2+0+0}{1\tfrac{1}{2}+0+0}=\tfrac{4}{3}$$

b. $\lim_{n\to\infty}\dfrac{2+5+9+13+...+4n}{2+6+11+16+...+5n}=\lim_{n\to\infty}\dfrac{1+5+9+13+...+4n+1}{1+6+11+16+...+5n+1}$

$$=\lim_{n\to\infty}\frac{\tfrac{1}{2}(n+1)(1+4n+1)}{\tfrac{1}{2}(n+1)(1+5n+1)}$$

$$=\lim_{n\to\infty}\frac{4n+2}{5n+2}=\lim_{n\to\infty}\frac{4+\frac{2}{n}}{5+\frac{2}{n}}=\frac{4+0}{5+0}=\tfrac{4}{5}$$

c. $\lim_{n\to\infty}\dfrac{1-2+4-8+...+(-2)^n}{2-4+8-16+...+-1\cdot(-2)^{n+1}}=\lim_{n\to\infty}\dfrac{1-2+4-8+...+(-2)^n}{2\cdot(1-2+4-8+...+(-2)^n)}=\lim_{n\to\infty}\tfrac{1}{2}=\tfrac{1}{2}$

d. $100+150+200+...+100+50n=\tfrac{1}{2}(n+1)(100+100+50n)$

$=(\tfrac{1}{2}n+\tfrac{1}{2})(200+50n)$

$=25n^2+125n+100$

$1+1{,}5+2{,}25+...+1{,}5^n=\dfrac{1-1{,}5^{n+1}}{1-1{,}5}=-2+2\cdot1{,}5^{n+1}=-2+2\cdot1{,}5^n\cdot1{,}5=3\cdot1{,}5^n-2$

$$\lim_{n\to\infty}\frac{100+150+200+...+100+50n}{1+1{,}5+2{,}25+...+1{,}5^n}=\lim_{n\to\infty}\frac{25n^2+125n+100}{3\cdot2^n-2}$$

$$=\lim_{n\to\infty}\frac{\frac{25n^2}{2^n}+\frac{125n}{2^n}+\frac{100}{2^n}}{3-\frac{2}{2^n}}=\frac{0+0+0}{3-0}=0$$

Opgave 65:

a. $400+300+225+186{,}75+126{,}5625+...=\dfrac{400}{1-0{,}75}=1600$

b. $1-\tfrac{1}{2}+\tfrac{1}{4}-\tfrac{1}{8}+\tfrac{1}{16}-\tfrac{1}{32}+...=\dfrac{1}{1--\tfrac{1}{2}}=\tfrac{2}{3}$

c. $S=\dfrac{40}{1-0{,}9}=400$

d. $S=\dfrac{500}{1-\tfrac{2}{3}}=1500$

Opgave 66:

$u_n = 200 \cdot 0{,}8^n$

$S = \dfrac{200}{1-0{,}8} = 1000\,\text{cm}$

Opgave 67:

a. $135 + 2 \cdot 135 \cdot 0{,}7 + 135 \cdot 0{,}7^2 = 390{,}15\,\text{cm}$

b. omlaag geldt: $135 + 135 \cdot 0{,}7 + 135 \cdot 0{,}7^2 + 135 \cdot 0{,}7^3 + ... = \dfrac{135}{1-0{,}7} = 450$

omhoog geldt: $135 \cdot 0{,}7 + 135 \cdot 0{,}7^2 + 135 \cdot 0{,}7^3 + 135 \cdot 0{,}7^4 + ... = \dfrac{135 \cdot 0{,}7}{1-0{,}7} = 315$

dus totaal: $450 + 315 = 765\,\text{cm}$

Opgave 68:

omlaag geldt: $40 + 40 \cdot 0{,}4 \cdot 0{,}75 + 40 \cdot (0{,}4 \cdot 0{,}7)^2 + 40 \cdot (0{,}4 \cdot 0{,}75)^3 + ... = \dfrac{40}{1-0{,}4 \cdot 0{,}75} = 57\tfrac{1}{7}$

omhoog geldt: $40 \cdot 0{,}4 + 40 \cdot 04{,}\cdot 0{,}4 \cdot 0{,}75 + 40 \cdot 0{,}4 \cdot (0{,}4 \cdot 0{,}75)^2 + 40 \cdot 0{,}4 \cdot (0{,}4 \cdot 0{,}75)^3 + ... =$

$\dfrac{40 \cdot 0{,}4}{1-0{,}4 \cdot 0{,}75} = 22\tfrac{6}{7}$

dus totaal: $57\tfrac{1}{7} + 22\tfrac{6}{7} = 80\,\text{m}$

Opgave 69:

a. $h_0 = 6$

$h_1 = 3\sqrt{2}$

$r = \dfrac{3\sqrt{2}}{6} = \tfrac{1}{2}\sqrt{2}$

$h_n = 6 \cdot (\tfrac{1}{2}\sqrt{2})^n$

$S_7 = \dfrac{6 - 6 \cdot (\tfrac{1}{2}\sqrt{2})^8}{1-\tfrac{1}{2}\sqrt{2}} = 19{,}2\,\text{cm}$

b. $S = \dfrac{6}{1-\tfrac{1}{2}\sqrt{2}} = 20{,}5\,\text{cm}$

c. iedere zijde wordt $\tfrac{1}{2}\sqrt{2}$ keer zo groot, dus de inhoud wordt $(\tfrac{1}{2}\sqrt{2})^3 = \tfrac{1}{4}\sqrt{2}$ keer zo groot

$I_n = 216 \cdot (\tfrac{1}{4}\sqrt{2})^n$

$S = \dfrac{216}{1-\tfrac{1}{4}\sqrt{2}} = 334{,}134\,\text{cm}^3$